Microeconomics
Part I, Paper 1
Consumer and Producer Theory; Rational Choice
Michaelmas Term 2016

Supervision set 3

(To be completed after Lecture 14)

1. Suppose that the price of good 1 decreases, while all other prices, and the consumer's income m stay the same. Assume that this good is ordinary (i.e., non-Giffen) but inferior for the consumer in question.

   (a) In order to explore the relationship between CV, CS and EV, graph the consumer's Marshallian demand function $x_1(p, m)$ where the price is depicted on the horizontal axis, and the demand depicted on the vertical axis. (Call the initial price p', and final price p''.) Does this curve have to be downward sloping or not? Explain.

   (b) At which price (p' or p'') does the consumer enjoy higher utility? Refer to these utility levels as u' and u'', respectively, and explain your answer.

   (c) What is Hicksian demand $h_1(p, u)$ about?

   (d) Illustrate $h_1(p, u')$ and $h_1(p, u'')$ on the graph. Do they intersect the Marshallian demand curve? If so, where and why? Also explain why you drew them the way you did. Would they look different if the good was normal?

   (e) Use the graph to give a ranking of CV, CS and EV associated with p' dropping to p''.

2. (a) Illustrate the isoquants of a production function $f(x, y)$ which has increasing returns to scale for small values of x and y, but has decreasing returns to scale for larger values of x and y.

   (b) Elaborate on why such a production function might be reasonable. Come up with a plausible story of a firm where we might expect its production function to behave this way.

3. Consider the following production function:

$$y = 2K^{1/2}L^{1/2}.$$

   (i) Comment on the returns to scale of this function.

   (ii) Represent the isoquants for $y = 4$ and $y = 8$.

   (iii) Suppose that one unit of L costs 1, and one unit of K costs 1. What is the cheapest way to produce 8 units of output?

   (iv) What is the equation for the long run output expansion path when factors' prices are both equal to 1? Show it on your isoquant graph. Show the short run output expansion path, and state your assumptions about the availability of factors.

   (v) Calculate the long-run total cost, average cost, and marginal cost functions when factors' prices are both equal to 1. Show the curves on the graph.

   (vi) Explain how the total cost, average cost, and marginal cost functions would change if the firm's technology were instead given by $y = (4KL)^{1/3}$, and show the new curves graphically.

4. Suppose that a firm's production function has the Leontief form $q = \min\{\frac{x}{2}, \frac{y}{3}\}$ where x and y are the two inputs.

   (i) Draw the isoquants.

   (ii) For a given level of output, identify the cost-minimising combination(s) of inputs on the diagram.

   (iii) Write down the cost function. Explain why in this example the slope of the isoquant is not relevant.

Additional questions

It is subject to the individual supervisor's discretion to assign any of the additional questions for their supervisions.

1. Suppose that a unit of output q can be produced using any of the following combinations of two inputs $(x,\ y)$: combination A (0.2, 0.5), combination B (0.3, 0.2) and combination C (0.5, 0.1).

   (i) Construct the isoquant for $q = 1$.

   (ii) Assuming constant returns to scale, construct the isoquant for $q = 2$.

   (iii) If the combination (0.25; 0.5) is also available, would it be included in the isoquant for $q = 1$?

2. (2004 Tripos)

   (i) Compare the returns to scale of the following technologies:

$$\begin{aligned} f(x,y) &= x^{0.4}y^{0.6} \\ g(x,y) &= x^{0.6}y^{0.8}. \end{aligned}$$

   (ii) Calculate the marginal rate of technical substitution for each. Briefly comment on your result.

3. A firm which uses two factors x_1 and x_2 has a production function of the following form:

$$y = x_1 + 3x_2$$

   (i) Draw the isoquants.

   (ii) Identify the cost-minimising combination(s) of inputs, and write down the corresponding cost function(s).

4. Let a firm's production function be given by $f(x,y) = (3\sqrt{x} + \sqrt{y})^2$. The price of input x is £3 and the price of input y is £1. What is the cheapest way to produce 16 units of output?

5. Consider the production technology

$$y = \min\{x_1 + 3x_2, 2x_3\}$$

   where x_1 is skilled labour, x_2 is unskilled labour and x_3 is capital. The wages of skilled and unskilled labour are w_1 and w_2 respectively, and the rental rate of capital is r. Identify the cost-minimising combination(s) of inputs, and write down the cost function(s).