1. As in the lectures, let $\tilde{k} = \frac{K}{AL}$ stand for capital per efficiency units of labour. The equation for evolution of capital is given by

$$\Delta k = \text{Savings} - (\delta + n + g)\tilde{k}$$

If all the capital income is saved and if capital earns its marginal product, then savings equals $MPK \cdot \tilde{k}$. Substituting this in the above equation we find that

$$\Delta k = MPK \cdot \tilde{k} - (\delta + n + g)\tilde{k}$$

In steady state:

$$MPK \cdot \tilde{k} = (\delta + n + g)\tilde{k}$$

$$\Rightarrow MPK\cdot = (\delta + n + g)$$

which means the economy is at its Golden rule steady state.

2. A straight-forward questions on endogenous growth.

   - Discuss the divergence between social and private return. The discussion should focus on the externalities of act of capital investment.

Derive the model:

$$Y = K^{\alpha}(A \cdot L)^{1-\alpha}$$

If the there are externalities associated with capital investment, we would have have

$$A = \lambda k$$

The rate of growth of the economy is given by

$$\frac{\Delta y}{y} = \frac{\Delta k}{k} = s \cdot \frac{y}{k} - (\delta + n)$$

$$= s \cdot \lambda^{1-\alpha} - (\delta + n)$$

Discuss the implication of the above given model.

(a) Non-convergence: low income countries do not necessarily grow faster.

(b) Growth rate depends on the saving rate of the economy

(c) Higher population growth, n, implies that slower growth in output her head leading to so called "poverty traps".

3. A question on endogenous growth again. The key relation is the complementarity between training and capital per worker in the economy.

$$Y = K^{\alpha}(T \cdot L)^{1-\alpha}$$

The per-worker production function for the economy is given by

$$y = T^{1-\alpha}\, k^{\alpha}$$

where T is the training that agents acquire. This is a function of the capital intensity in the economy.

$$T = \beta k$$

This basically means that as capital intensity in the economy increases, the worker are required to be more trained / skilled to work with the capital.

$$Y = K^{\alpha}(T \cdot L)^{1-\alpha}$$

The growth rate of output per worker y and capital per worker k is given by

$$\begin{aligned}\frac{\Delta y}{y} = \frac{\Delta k}{k} &= s \cdot \frac{y}{k} - (\delta + n) \\ &= s\beta^{1-\alpha} - (\delta + n)\end{aligned}$$

Implications are that that

i. The growth rate of y and k increases with β. That means the greater the "Productivity-effect", the higher the β, the faster the economy grows.

ii. The growth rate of y and k increases with saving rate s.

iii. The growth rate of y and k decreases with n and δ.