1. We get the steady state condition by equating the growth rate of $\tilde{k}$ equation to zero.

$$\frac{\Delta\tilde{k}}{\tilde{k}} = s \cdot \frac{f(\tilde{k})}{\tilde{k}} - (\delta + n + g) = 0$$

From this we can obtain $\tilde{y}^*$, the steady state value of income per effective worker

$$\frac{\tilde{y}^*}{\tilde{k}^*} = \frac{f(\tilde{k}^*)}{\tilde{k}^*} = \frac{(\delta + n + g)}{s} \tag{SS}$$

We need to establish the following facts before we tackle the question

- A is a constant at a given point in time. Thus $\tilde{y}$ and y will move in the same direction. So if some change increases $\tilde{y}$ it will increase y as well.
- With a cross-sectional analysis we assume that most of the countries in the sample are in their respective steady states. It is reasonable to assume that the small proportion of countries that are not in steady state have a statistically insignificant effect on the results.

a) From (SS) it is clear that an increase in the saving rate s would lead to an fall in output-capital ratio $\frac{\tilde{y}^*}{\tilde{k}^*}$. A fall in $\frac{\tilde{y}^*}{\tilde{k}^*}$ implies a rise in $\tilde{y}^*$ and y^*. Thus, s and y^* have a *positive* relationship.

b) From (SS), we find that a rise in n leads to a rise in $\frac{\tilde{y}^*}{\tilde{k}^*}$ which implies a fall in y^*. Thus, n and y^* has a *negative* relationship.

The answer, of course, would change if we were looking at the relationship between rate of growth of output per head y and s and n. The growth rate of output per worker in the economy depends only on rate of technological progress g. s and n do not affect the levels and the convergence dynamics. But they do not affect the steady state growth rates. Thus, we would not expect that growth rate of y would have any relation with s and n respectively.

2.

$$\frac{\partial Y}{\partial K} = \alpha \left(\frac{AL}{K}\right)^{1-\alpha} = (r+\delta) \tag{1}$$

$$\frac{\partial Y}{\partial L} = (1-\alpha) \left(\frac{K}{AL}\right)^{\alpha} = w \tag{2}$$

Using (1) we obtain the share of capital income as proportion of total income in the economy

$$\frac{(r+\delta)\cdot K}{Y} = \frac{\alpha \left(\frac{AL}{K}\right)^{1-\alpha} \cdot K}{K^{\alpha}(AL)^{1-\alpha}} = \alpha$$

Similarly, using (2) we obtain the share of labour income as proportion of total income in the economy

$$\frac{w \cdot K}{Y} = \frac{(1-\alpha) \left(\frac{K}{AL}\right)^{\alpha} \cdot L}{K^{\alpha}(AL)^{1-\alpha}} = (1-\alpha)$$

Intuition: We know that the capital's share of income = $MPK \cdot \frac{K}{Y}$. In the steady state, as we have discussed in the lectures, the capital-output ratio $\frac{K}{Y}$ is constant. We also know that *MPK* can be written as a function of $\tilde{k}$, which is constant in the steady state. Therefore, *MPK* itself must be a constant. Thus, capital's share of income is constant. Labour's share $= 1-$ [capital's share]. Hence, if capital's share is constant, we see that labour's share is of income is also constant.

3. a) Using the dynamic equation for change of capital we can find the steady state value of $\tilde{y}$ as a function of s, n, g and δ.

$$\frac{\Delta \tilde{k}}{\tilde{k}} = s \cdot \frac{f(\tilde{k})}{\tilde{k}} - (\delta + n + g) = 0$$

The production function can be written as $\tilde{y}^2 = \tilde{k}$. Plugging this production function into the above steady state condition, we find the steady state value of $\tilde{y}$:

$$\tilde{y}^* = \frac{s}{\delta + n + g} \tag{3}$$

b) Using (3), we find values for y each country.

Developed Country: $\tilde{y} = \frac{0.28}{0.04+0.01+0.02} = 4$
Less-developed country: $\tilde{y} = \frac{0.10}{0.04+0.04+0.02} = 1$

c) The equation for $\tilde{y}^*$ that we derived in part (a) shows that the less-developed country could raise its level of income by reducing its population growth rate n or by increasing its saving rate s. Policies that reduce the population growth include introducing methods of birth control and implementing disincentives for having children. Policies that increase the saving rate include increasing public saving by reducing the budget deficit and introducing private saving incentives such as tax concessions and increase return to saving.