1. a) Product rule: $dz = x\,dy + y\,dx$
   To get growth rates, divide by z on the LHS and $x \cdot y$ on the RHS
$$\frac{dz}{z} = \frac{dx}{x} + \frac{dy}{y}$$
   b) Similarly
$$\frac{dz}{z} = \frac{dx}{x} - \frac{du}{u} - \frac{dv}{v}$$

2. First order conditions of the firm's profit maximisation problem give us the demand for factors
$$\frac{\partial \Pi}{\partial K} = F_K(K, L) - r = 0$$
$$\frac{\partial \Pi}{\partial L} = F_L(K, L) - w = 0$$
   Now for the Cobb Douglas function $Y = K^\alpha (AL)^{1-\alpha}$ (with *labour augmenting technological progress*), we have:
$$F_K(K, L) = \alpha (AL)^{1-\alpha} K^{\alpha-1}$$
$$F_L(K, L) = (1-\alpha) A^{1-\alpha} K^\alpha L^{-\alpha}$$
   Therefore, the demand for factors is
$$K^d = AL \left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}$$
$$L^d = A^{\frac{1-\alpha}{\alpha}} K \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}}$$
   A increases both demands, with elasticity 1 and $\frac{1-\alpha}{\alpha}$ respectively, because it makes factors more productive.

3. i. Assume K and L are fixed. Then we have:
$$\frac{\bar{K}}{\bar{L}} = g(r, A) = A \left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}$$
$$\frac{\bar{L}}{\bar{K}} = f(w, A) = A^{\frac{1-\alpha}{\alpha}} \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}}$$

Since $g_r < 0, f_w < 0, g_A > 0, f_A > 0$, it follows that if A goes up, so do r and w.

ii. Now assume that the capital is infinitely elastic. This implies that $r = \bar{r}$. We then have

$$\frac{K}{\bar{L}} = g(\bar{r}, A)$$
$$\frac{\bar{L}}{K} = f(w, A)$$

Given the above partial derivatives, it follows that if A goes up, so does w but r remains fixed at $\bar{r}$.