# Economics 2: Growth (Endogenous Growth)

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Lecture 7, Week 9

# Solow - III

Cobb Douglas production function

$$Y == K^{\alpha}(AL)^{1-\alpha}$$

Per-worker production function

$$y = A^{1-\alpha}k^{\alpha}$$

Determining the growth rate of $y$

$$\frac{\Delta y}{y} = (1-\alpha)\frac{\Delta A}{A} + \alpha\frac{\Delta k}{k}$$

# Growth rates in Solow -III

$$\frac{\Delta y}{y} = (1-\alpha)\frac{\Delta A}{A} + \alpha\frac{\Delta k}{k}$$

1. $\frac{\Delta A}{A} = g$
2. $\frac{\Delta k}{k} = g$ (steady state)

$$\Rightarrow \quad \frac{\Delta y}{y} = (1-\alpha)\, g + \alpha\, g = g$$

- $y$ grows at rate $g$ in steady state because
  - $A$ always grows at the rate $g$
  - $k$ grows at the rate $g$ in steady state

# Externalities of Investment

Till now we have assumed that $A$ grows at an exogenous rate $g$

## Assumption (Positive Externalities of Investment)

*The act of investment generates new ideas for both the investing firm and other firms in the economy. Specifically,*

$$A = \lambda k \qquad (\lambda > 0)$$

*stock of knowledge $A$ is proportional to the stock of capital stock per worker $k$*

- Investing ($k \uparrow$) affects output $y$ through two distinct channels:
  - **Direct effect**: greater capital stock per worker lead to greater output per worker
  - **Indirect effect**: higher capital stock per worker leads to higher value $A$ which leads to higher output per worker.

# The Two Channels

- Investing ($k \uparrow$) affects output $y$ through two distinct channels:
  - **Direct effect**: higher $k$ leads to higher $y$
  - **Indirect effect**: higher $k$ leads to higher value of $A$ which leads to higher $y$.

$$
\begin{aligned}
y &= (A)^{1-\alpha}\, k^{\alpha} \\
&= (\lambda k)^{1-\alpha}\, k^{\alpha} \\
&= \lambda^{1-\alpha}\, k
\end{aligned}
$$

$$\Rightarrow \quad \frac{y}{k} = \lambda^{1-\alpha} = \text{ constant}$$

$$\Rightarrow \quad \frac{\Delta y}{y} = \frac{\Delta k}{k}$$

# Deriving Endogenous Growth

$$\frac{\Delta K}{K} = s \cdot \frac{Y}{K} - \delta$$

$$\Rightarrow \quad \frac{\Delta k}{k} = s \cdot \frac{y}{k} - (\delta + n)$$

$$= s \cdot \lambda^{1-\alpha} - (\delta + n)$$

- growth rate of $k$ depends on the $s, \delta, n$ and $\lambda$

# Endogenous Growth

$$\Rightarrow \quad \frac{\Delta y}{y} = s \cdot \lambda^{1-\alpha} - (\delta + n)$$

Perpetual growth of $k$ and $y$ if $s \cdot \lambda^{1-\alpha} > (\delta + n)$

- New Results:
  1. Steady state can only be defined in terms of growth rates. It cannot be defined in terms of levels anymore.
  2. Growth rate of $y$ and $k$ depends on $\delta, n, s$ and crucially on $\lambda$
     - Countries with higher $\lambda$ grow faster. Explains why developed economies keep growing faster than certain under-developed. (due to higher $\lambda$)
     - Lower saving rate can lead to lower growth
     - Higher population growth rate can lead to slower growth. Paul Romer attributes slowdown in US growth in the 60s to this effect.

# Factor Prices

For Cobb-Douglas function: $\frac{(r+\delta)K}{Y} = \alpha$

$$r = \frac{\partial Y}{\partial K} - \delta = \alpha \frac{Y}{K} - \delta$$
$$= \alpha \frac{y}{k} - \delta = \alpha \lambda^{1-\alpha} - \delta$$

- $r$ is a constant in the economy

Similarly given that $\frac{w \cdot L}{Y} = 1 - \alpha$

$$w = (1-\alpha) y = (1-\alpha)\lambda^{1-\alpha} k$$

- $w$ grows at the same rate a $k$, i.e. at the economy wide growth rate $g$.

# Summary: Endogenous Growth

- Externalities of capital investment create an extra channel through which investment affects the output
- Technological Progress is endogenised
  - Stock of Knowledge $A$ is proportional to capital stock per worker $k$
- We get perpetual growth of $k$ and $y$ which depends on
  - $s, \delta, n$
  - the strength of externality of capital investment, namely the value of $\lambda$
  - higher the value of $\lambda$, the faster the economy grows
- $r$ is constant and $w$ grows at the rate at which $k$ and $y$ grows