Economics 2: Growth
(Golden Rule)

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Lecture 6, Week 9

Effective Units of Labour

- AL: effective units of labour
- $\tilde{k}_t = \frac{K}{AL}$: capital stock per effective unit of labour
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- Fundamental Equation - III

$$\frac{\Delta \tilde{k}_t}{\tilde{k}_t} = s\frac{\tilde{y}_t}{\tilde{k}_t} - (\delta + n + g)$$

In convergence dynamics, saving does not exactly offset the reduction in $\tilde{k}_t$ attributable to depreciation, population growth and technological progress.

- Growth rate of $\tilde{k}_t$ (and $\tilde{y}_t$) determined by s, δ, n and g.

Steady State

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$$\frac{\tilde{y}_t}{\tilde{k}_t} = \frac{\delta + n + g}{s}$$

In steady state, saving $sf(\tilde{k}_t)$ *exactly offsets the reduction in* $\tilde{k}_t$ *attributable to depreciation, population growth and technological progress.*

- Level of $\tilde{k}_t$ (and $\tilde{y}_t$) determined by s, δ, n and g

Growth: Steady State vs Convergence Dynamics

$$\frac{\Delta \tilde{k}_t}{\tilde{k}_t} = s\frac{\tilde{y}_t}{\tilde{k}_t} - (\delta + n + g) \qquad \text{(Convergence dynamics)}$$

$$\frac{\Delta \tilde{k}_t}{\tilde{k}_t} = s\frac{\tilde{y}_t}{\tilde{k}_t} - (\delta + n + g) = 0 \qquad \text{(Steady State)}$$

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$$\frac{\Delta \tilde{k}_t}{\tilde{k}_t} = \frac{\Delta k}{k} - g = 0 \qquad \Rightarrow \quad k \text{ grows at the rate } g$$

$$\frac{\Delta \tilde{y}_t}{\tilde{y}_t} = \frac{\Delta y}{y} - g = 0 \qquad \Rightarrow \quad y \text{ grows at } g$$

Social Welfare Maximization

$\tilde{k}^*$ determined by s, n, δ and g.

$$\frac{f(\tilde{k}^*)}{\tilde{k}^*} = \frac{(\delta + n + g)}{s}$$

Any steady state (any $\tilde{k}^*$) can be reached with the right combination of s, n, δ and g

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Which one is the best social welfare maximising steady state

What social welfare maximization mean?

Social Welfare Maximization

- **Simple rule**: Consider consumption per head in steady state

$$\begin{aligned}\tilde{c}^*(\tilde{k}^*) &= \quad \tilde{y}^* - s \cdot \tilde{y}^* \\ &= f(\tilde{k}^*) - s \cdot f(\tilde{k}^*)\end{aligned}$$

- Intertemporal choice:

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$$C \uparrow \Rightarrow S \downarrow \Rightarrow I \downarrow \Rightarrow \tilde{k}^* \downarrow \Rightarrow C \downarrow$$

Golden Rule

Definition (Golden Rule Steady-State)

$$\max_{\tilde{k}^*} \quad \tilde{c}^*(\tilde{k}^*)$$

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$$\tilde{c}^* \equiv \frac{C^*}{AL} = f(\tilde{k}^*) - s \cdot f(\tilde{k}^*)$$
$$= f(\tilde{k}^*) - (\delta + n + g) \cdot \tilde{k}^*$$

Golden Rule

$$\max_{\tilde{k}^*} \ \tilde{c}^*(\tilde{k}^*)$$

$$\Rightarrow \qquad f'(\tilde{k}^*) - \delta = n + g$$

Net marginal product of capital = Growth rate of capital/output

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Net marginal product of capital = Growth rate of capital/output

- Real rate of interest should be equal to the real rate of growth of the economy
- If capital stock below the golden rule level, the government should encourage saving and investment in the economy

$$\begin{aligned} Y &= F(K, AL) \\ &= K^{\alpha}(AL)^{1-\alpha} \qquad \text{(Cobb-Douglas)} \end{aligned}$$

Y grows for three reasons:

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2. Growth in L
3. Technological Progress (Growth in A)

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Y grows for three reasons:

1. Growth in K
2. Growth in L
3. Technological Progress (Growth in A)
   - Total Factor Productivity Growth (TFPG)
   - very difficult to measure

$$
\frac{\Delta Y}{Y} = \alpha\frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L} + \underbrace{(1-\alpha)\frac{\Delta A}{A}}_{\text{TFPG}}
$$

Capital income share

Proposition

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- Share of capital income: $\dfrac{(r+\delta)\cdot K}{Y}$
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  - $\frac{K}{Y}$ is constant (steady state property)
  - $r+\delta = f'(\tilde{k}_t)$ is constant (steady state property)

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◇ **Tutorial 17**: Show that for a Cobb Douglas production function

$$\frac{r + \delta \cdot K}{Y} = \alpha$$

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For Cobb-Douglas production function $Y = K^{\alpha}(AL)^{1-\alpha}$, the share of capital income and labour income is α and $(1-\alpha)$ respectively

- **Tutorial 17**: Show that for a Cobb Douglas production function

$$\frac{r + \delta \cdot K}{Y} = \alpha$$

  - $\dfrac{(r+\delta) \cdot K}{Y}$ is easy to measure.
  - For most developed economies, $\alpha \approx \frac{1}{3}$.

Total Factor Productivity

Definition (Total Factor Productivity Growth)

Growth of output that cannot be explained by growth of inputs. It is also called the Solow residual because it measures the residual growth and was first measured by Solow in 1957.

$$\text{TFPG} = \frac{\Delta Y}{Y} - \left(\frac{1}{3} \cdot \frac{\Delta K}{K} - \frac{1}{3} \cdot \frac{\Delta L}{L} \right)$$

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- **US**: TFPG accounts for one-third of the growth
- **UK**: TFPG accounts for half of the growth

Summary

Two things are less than satisfactory with Solow Growth model

1. TFP is exogenous
   - we cannot explain exactly why we get growth in steady state
   - it does not tell us how to encourage growth
   - e.g. cannot explain the slowdown in the 70s

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⊙ Solow Model just tells us that we can solve the growth conundrum by looking for answers in technological progress.