Economics 2: Growth (Solow Model -III: Technology)

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Lecture 5, Week 8

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- Empirically we observe that the output per worker and capital stock per worker grows at a positive rate which Solow - II cannot explain.

Solow Model - III

Definition (Solow Model III)

Solow model with positive population growth and technological progress.

Assumption

a) *Positive population growth* $\Rightarrow \frac{\Delta L}{L} = n > 0$

b) *Positive technological progress* $\Rightarrow \frac{\Delta A}{A} = g > 0$

Technology in Solow Growth Model

Definition (Labour-augmenting Technology)

$$Y = F(K, AL)$$

- *technological progress occurs when A increases over time*
- *a unit of labour becomes more productive with technological progress (as A increases)*

⊙ What happens to the production function as A increases?

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- We express all variables in terms of effective units.

$$\tilde{y}_t \equiv \frac{Y}{AL}$$

$$\tilde{k}_t \equiv \frac{Y}{AL}$$

Growth Rates

- Using constant returns to scale (CRS)

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$$= \frac{\Delta K}{K} - n - g$$

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$$\frac{\Delta K}{K} = \frac{\Delta \tilde{k}_t}{\tilde{k}_t} + n + g$$

Solow - III: Deriving the Fundamental Equation

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$$\frac{\Delta K_t}{K_t} = s \cdot \frac{Y_t}{K_t} - \delta K_t \quad (1)$$

$$\frac{\Delta \tilde{k}_t}{\tilde{k}_t} = \frac{\Delta K_t}{K_t} - n - g \quad (2)$$

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$$\frac{\Delta \tilde{k}_t}{\tilde{k}_t} = s\,\frac{\tilde{y}_t}{\tilde{k}_t} - (\delta + n + g) \tag{FE-III}$$

Fundamental Equation - III

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$$\frac{\Delta \tilde{k}_t}{\tilde{k}_t} = s \cdot \frac{\tilde{y}_t}{\tilde{k}_t} - (\delta + n + g)$$

- *The growth rate of k_t depends*
  - *positively on s*
  - *positively on $\frac{Y_t}{K_t}$*
  - *negatively on δ*
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Solow - III: Steady State

Definition (Steady State Condition)

$$\frac{\Delta k_t}{k_t} = s \cdot \frac{\tilde{y}_t}{\tilde{k}_t} - (\delta + n + g) = 0$$

$$\left[\frac{\tilde{y}^*}{\tilde{k}^*}\right] = \frac{\delta + n + g}{s}$$

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Steady-State Growth

$$\tilde{k}_t = \tilde{k}^*$$

- Capital per effective worker $\tilde{k}_t$ is constant

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- Capital per worker k grows at the rate g

$$\frac{\Delta k}{k} = g$$

Steady State Growth Path

- Similarly, output per effective worker $\tilde{y}_t$ is constant

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- We have finally got growth for k and y in steady state
  - Without technological progress, capital accumulation runs into diminishing returns
  - With technological progress, improvements in technology continually offsets the diminishing returns to capital accumulation

Solow - III: Convergence Dynamics

Proposition (Convergence Dynamics of Solow - II)

$$\frac{\Delta \tilde{k}_t}{\tilde{k}_t} = s\,\frac{\tilde{y}_t}{\tilde{k}_t} - (\delta + n + g)$$

$$= s\left(\frac{\tilde{y}_t}{\tilde{k}_t} - \left[\frac{\tilde{y}^*}{\tilde{k}^*}\right]\right)$$

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- *Further the economy is from the steady state, faster the growth rate of capital per worker k*
- *Higher the saving rate s, faster the economy converges to the steady state*

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- Empirically we observe variation in growth rate of output per worker across countries which Solow -III cannot explain very well.
- It does tell us where to look for an explanation . . . . . .