Economics 2: Growth (Solow Model II & III)

© Kumar Aniket

Lecture 4, Week 7

Definition (Solow Model II)

The most basic Solow model with positive population growth and no technological progress.

Definition (Solow Model II)

The most basic Solow model with positive population growth and no technological progress.

Assumption

a) *Positive population growth* $\Rightarrow \frac{\Delta L}{L} = n > 0$

Solow Model - II

Definition (Solow Model II)

The most basic Solow model with positive population growth and no technological progress.

Assumption

a) *Positive population growth* $\Rightarrow \frac{\Delta L}{L} = n > 0$

b) *no technological progress* $\Rightarrow \frac{\Delta A}{A} = 0$

CRS Production Function

$$Y = F(K, L)$$

Constant Returns to Scale Production Function

$$\lambda Y = F(\lambda K, \lambda L)$$

where $\lambda = \frac{1}{L}$

Per Worker Production Function

$$y = f(k)$$

where $y = \frac{Y}{L},\ k = \frac{K}{L}$

Growth Rate of Capital Per Worker

$$k_t = \frac{K_t}{L_t}$$

$$\frac{\Delta k_t}{k_t} = \frac{\Delta K_t}{K_t} - \frac{\Delta L_t}{L_t}$$

$$gr(k_t) = gr(K_t) - gr(L_t)$$

Growth Rate of Capital Per Worker

$$k_t = \frac{K_t}{L_t}$$

$$\frac{\Delta k_t}{k_t} = \frac{\Delta K_t}{K_t} - \frac{\Delta L_t}{L_t}$$

$$gr(k_t) = gr(K_t) - gr(L_t)$$

$$\frac{\Delta K_t}{K_t} = \frac{\Delta k_t}{k_t} + \frac{\Delta L_t}{L_t}$$

$$= \frac{\Delta k_t}{k_t} + n$$

Solow - II: Deriving the Fundamental Equation

$$S = I$$

$$sY_t = \Delta K_t + \delta K_t$$

$$\Rightarrow \quad \frac{\Delta K_t}{K_t} = s\frac{Y_t}{K_t} - \delta \qquad \text{(rearranging)}$$

$$\frac{\Delta L_t}{L_t} + \frac{\Delta k_t}{k_t} = s\frac{Y_t}{K_t} - \delta \qquad \text{(substituting)}$$

$$\frac{\Delta k_t}{k_t} = s\frac{y_t}{k_t} - (\delta + n)$$

Fundamental Equation - II

Definition (Fundamental Equation - II)

$$\frac{\Delta k_t}{k_t} = s \cdot \frac{y_t}{k_t} - (\delta + n)$$

- *The growth rate of k_t depends*
  - *positively on s*
  - *positively on $\frac{Y_t}{K_t}$*
  - *negatively on δ*

Fundamental Equation - II

Definition (Fundamental Equation - II)

$$\frac{\Delta k_t}{k_t} = s \cdot \frac{y_t}{k_t} - (\delta + n)$$

- *The growth rate of k_t depends*
  - *positively on s*
  - *positively on $\frac{Y_t}{K_t}$*
  - *negatively on δ*
  - *negatively on n*

Solow - II: Steady State

Definition (Steady State Condition)

$$\frac{\Delta k_t}{k_t} = s \cdot \frac{y_t}{k_t} - (\delta + n) = 0$$

$$\left[\frac{y_t^*}{k_t^*}\right] = \frac{\delta + n}{s}$$

Solow - II: Steady State

Definition (Steady State Condition)

$$\frac{\Delta k_t}{k_t} = s \cdot \frac{y_t}{k_t} - (\delta + n) = 0$$

$$\left[\frac{y_t^*}{k_t^*}\right] = \frac{\delta + n}{s}$$

- *The steady-state Output and Capital stock levels are*
  - *positively related with s*
  - *negatively related with δ*
  - *negatively related with n*

Solow - II: Steady State

- Steady State: $y^* = f(k^*)$
  - $\text{gr}(k) = 0$

    $\Rightarrow$ Capital Stock per worker in the economy is constant
  - $\text{gr}(K) = n$

    $\Rightarrow$ Capital Stock grows at the rate n
  - $\text{gr}(y) = 0$

    $\Rightarrow$ Output per worker in the economy is constant
  - $\text{gr}(Y) = n$

    $\Rightarrow$ Output grow at the rate n

Factor Prices

Steady State factor Prices

$$r = f'(k)$$

$$w = f(k) - kf'(k)$$

Factor Prices

Steady State factor Prices

$$r = f'(k)$$

$$w = f(k) - kf'(k)$$

- with $k = k^*$, the steady state factor prices remain constant.

Factor Prices

Steady State factor Prices

$$r = f'(k)$$

$$w = f(k) - kf'(k)$$

- with $k = k^*$, the steady state factor prices remain constant.
- Kaldor Facts state that
  - ✓ r is constant
  - X w is growing at a constant rate

Solow - II: Convergence Dynamics

Proposition (Convergence Dynamics of Solow - II)

$$\frac{\Delta k_t}{k_t} = s\,\frac{y_t}{k_t} - (\delta + n)$$

$$= s\left(\frac{y_t}{k_t} - \left[\frac{y_t^*}{k_t^*}\right]\right)$$

Solow - II: Convergence Dynamics

Proposition (Convergence Dynamics of Solow - II)

$$\frac{\Delta k_t}{k_t} = s\,\frac{y_t}{k_t} - (\delta + n)$$

$$= s\left(\frac{y_t}{k_t} - \left[\frac{y_t^*}{k_t^*}\right]\right)$$

- *Further the economy is from the steady state, faster the growth rate of capital per worker k*

Proposition (Convergence Dynamics of Solow - II)

$$\frac{\Delta k_t}{k_t} = s\,\frac{y_t}{k_t} - (\delta + n)$$
$$= s\left(\frac{y_t}{k_t} - \left[\frac{y_t^*}{k_t^*}\right]\right)$$

- *Further the economy is from the steady state, faster the growth rate of capital per worker k*
- *Higher the saving rate s, faster the economy converges to the steady state*

Summary

- Stationary state is determined by s, δ and n
  - a higher $s \Rightarrow$ a higher k^* and y^*
  - a higher $\delta \Rightarrow$ a lower k^* and y^*
  - a higher $n \Rightarrow$ a lower k^* and y^*
- Solow Model - I says that poor countries are poor because

Summary

- Stationary state is determined by s, δ and n
  - a higher s $\Rightarrow$ a higher k^* and y^*
  - a higher δ $\Rightarrow$ a lower k^* and y^*
  - a higher n $\Rightarrow$ a lower k^* and y^*
- Solow Model - I says that poor countries are poor because
  1. their depreciation rate δ is high (unlikely)
  2. their saving rates s are low (unlikely)
  3. their level of technology is low (most likely)
  4. their population growth is high (fairly likely)

Summary

- With positive population growth, the model predicts that economy's
  - √ capital stock and output grow at the rate n
  - X capital stock per worker and output per worker do not grow

Summary

- With positive population growth, the model predicts that economy's
  - √ capital stock and output grow at the rate n
  - X capital stock per worker and output per worker do not grow
- Solow - II gives us steady state growth of capital stock and output
- Empirically we observe that the output per worker and capital stock per worker grows at a positive rate which Solow - II cannot explain.