# Economics 2: Growth (Growth in the Solow Model)

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Lecture 3, Week 7

# Solow Model - I

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b) *no technological progress* $\Rightarrow \frac{\Delta A}{A} = 0$

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- *the fundamental equation*
  - *derived from the saving investment equality*
  - *would change to account for population growth*
  - *would change to account for technological progress*

# Growth Rate of Capital Stock

$$\Delta K_t = sY - \delta K_t$$

$$\frac{\Delta K_t}{K_t} = s\frac{Y_t}{K_t} - \delta$$

- growth rate of capital
  - increases with the saving rate $s$

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  - decrease with rate of depreciation $\delta$
  - increases with output-capital ratio $\frac{Y_t}{K_t}$
    - $\frac{Y_t}{K_t}$ decreases as $K_t$ increases (Worksheet 2, Figure 1)

# Stationary State

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*In Solow Model - I*

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$$\frac{\Delta K_t}{K_t} = s\frac{Y_t}{K_t} - \delta = 0$$

$$\left[\frac{Y_t^*}{K_t^*}\right] = \frac{\delta}{s}$$

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- Does this match our observation of the world?

# Worksheet 3

- Illustrate the effect of following changes on the stationary state variables $K^*$ and $Y^*$
  - increase in depreciation rate
  - decrease in saving rate
  - technological progress
- Does this satisfactorily explain why some countries remain poor?

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- Assume: $K_t < K_t^*$
  - as $K_t \uparrow$, capital-output ratio $\downarrow$
  - growth rate of capital is the difference between current capital-output ratio and stationary state capital-output ratio
  - the further away from stationary state the economy is, the faster the rate at which capital grows
  - the further away from stationary state the economy is, the faster the rate at which output grows

# Summary

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  2. their saving rates $s$ are low (unlikely)
  3. their level of technology is low (most likely)

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- explains the take - off phase of growth
  - Germany and Japan in 30 years after World War II
  - When reform raises factor productivity i.e. China, India

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Puzzle: According to Solow Model - I economic growth (of $K$ and $Y$) can only be achieved if the economy is not in stationary state. Once it reaches stationary state, there is no growth.

- This obviously contradicts our observation of the world around us
- We need to enrich the model with population growth and technological progress to see if it can provide us with a better explanation.