Economics 2: Growth
Production Function & Kaldor Facts

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Lecture 1, Week 6

Long-run Growth: Point of Departure ...

- Ignore the Demand Side
  - Assumption: Prices are flexible
  - Assumption: Agents form correct expectations
- Carefully specify the supply side
  - Labour is exogenous given
  - Capital is endogenous over time
  - Technology exogenous to start with ...

The Production Function

$$Y = F(K,L) \quad \text{where} \quad \begin{aligned} Y &= \text{output} \\ K &= \text{capital (input / factor)} \\ L &= \text{labour (input / factor)} \end{aligned}$$

- Assumptions:
  - Constant Returns to Scale
  - Marginal Products positive and diminishing

Marginal Products

- Marginal Product of Labour:

$$\begin{aligned} \frac{\partial Y}{\partial L} &= F_L > 0 \quad \textit{positive} \\ \frac{\partial^2 Y}{\partial L^2} &= F_{LL} < 0 \quad \textit{and diminishing} \end{aligned}$$

- Marginal Product of Capital:

$$\begin{aligned} \frac{\partial Y}{\partial K} &= F_K > 0 \quad \textit{positive} \\ \frac{\partial^2 Y}{\partial K^2} &= F_{KK} < 0 \quad \textit{and diminishing} \end{aligned}$$

Constant Returns to Scale

$$\lambda Y = F(\lambda K, \lambda L) \qquad \lambda > 0$$

- Implication of Constant Returns to Scale

$$F_{KL} > 0 \qquad \textit{(Factors are Complementary)}$$

$$Y = F_K K + F_L L \quad \textit{(Euler's Theorem)}$$

- Euler's Theorem $\Rightarrow$ Factor payments exhaust the output

Constant Return to Scale → Representative Firm

- Size does not matter
  - The whole country could be one firm
  - Alternative, the country could be divided into infinite number of tiny firms
- We try to understand the economy by understanding the action of a single *representative* firm

Cobb-Douglas Production Function

$$Y = AK^{\alpha}L^{1-\alpha}$$

- A: constant (represents state of technology)
- A plays a key role in theory of growth
- check
  - Exhibits constant returns of scale?
  - Factors are complementary?
  - Euler's theorem is satisfied?

Factor Markets

- Factor Supply Fixed (inelastic):

$$L^s = \overline{L}$$
$$K^s = \overline{K}$$

- Factor Demand determined by firms
- Factor prices determined by the demand and supply for factors
  - factor prices effectively determined by the demand

Profit Maximizing Firm

$$\Pi = F(K,L) - rK - wL$$

- First Order Conditions

$$\frac{\partial \Pi}{\partial K} = F_K(K,L) - r = 0$$

$$\frac{\partial \Pi}{\partial L} = F_L(K,L) - w = 0$$

- Demand for Factors

$$K^d = L \cdot g(r) \qquad g'(r) < 0$$

$$L^d = K \cdot h(w) \qquad h'(w) < 0$$

Factor Prices

- Equate Demand and Supply

$$\begin{aligned}\bar{K} &= K^d \\ &= \bar{L}g(r)\end{aligned}$$

$$\begin{aligned}\bar{L} &= L^d \\ &= \bar{K}h(w)\end{aligned}$$

- As the exogenous capital-labour ratio increases:

$$\left(\frac{\bar{K}}{\bar{L}}\right)\uparrow \;\Rightarrow \begin{cases}\downarrow r \\ \uparrow w\end{cases}$$

One-worker Production Function

$$\lambda Y = F(\lambda K, \lambda L)$$

$$\frac{Y}{L} = F\left(\frac{K}{L}, 1\right) \qquad \lambda = \frac{1}{L}$$

$$y = f(k) \qquad y = \frac{Y}{L}; k = \frac{K}{L}$$

- refer to the Worksheet 1

Facts of Growth: The Kaldor Facts

- $\frac{K}{L}$ grows at constant rate
- r is constant
- $\frac{Y}{K}$ is constant
- w grows at a constant rate
- Growth rate & Income levels vary substantially across countries
- Growth rate not necessarily constant over time