Economic Growth: The Neo-classical & Endogenous Story

EC307 Economic Development

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University of Cambridge & LSE Summer School

Lecture 2

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Readings

Tables and figures in this lecture are taken from:

Chapters 3 & 4 of Ray (1998).

Aniket's Lecture Notes on Economic Growth.

Lucas Jr, R.E. (1990). Why Doesn't Capital Flow from Rich to Poor Countries? *American Economic Review*. 80(2): 92-96.

Jones, C. (1997). On The Evolution of the World Income Distribution. *Journal of Economic Perspectives*, vol. 11, pp. 19-36.

- **Class based on** Jones, B. & Olken, B. (2005). Do Leaders Matter? National Leadership and Growth since World War II. *Quarterly Journal of Economics*, 120(3):835–864.

♣ `Further Reading:` Lucas, R. E. (2004). *Lectures on Economic Growth.* Harvard University Press

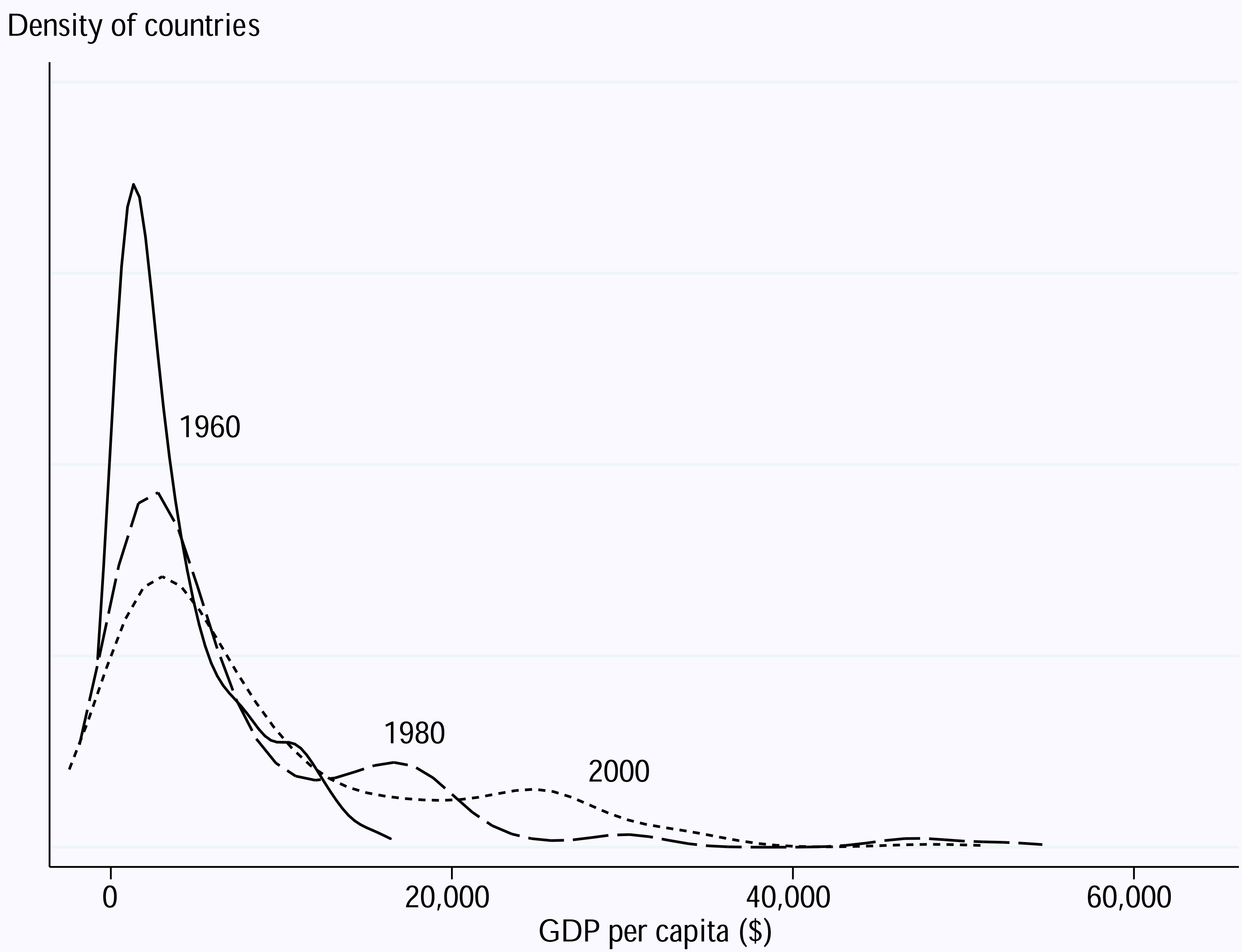


FIGURE 1.1 Estimates of the distribution of countries according to PPP-adjusted GDP per capita in 1960, 1980, and 2000.

Density of countries

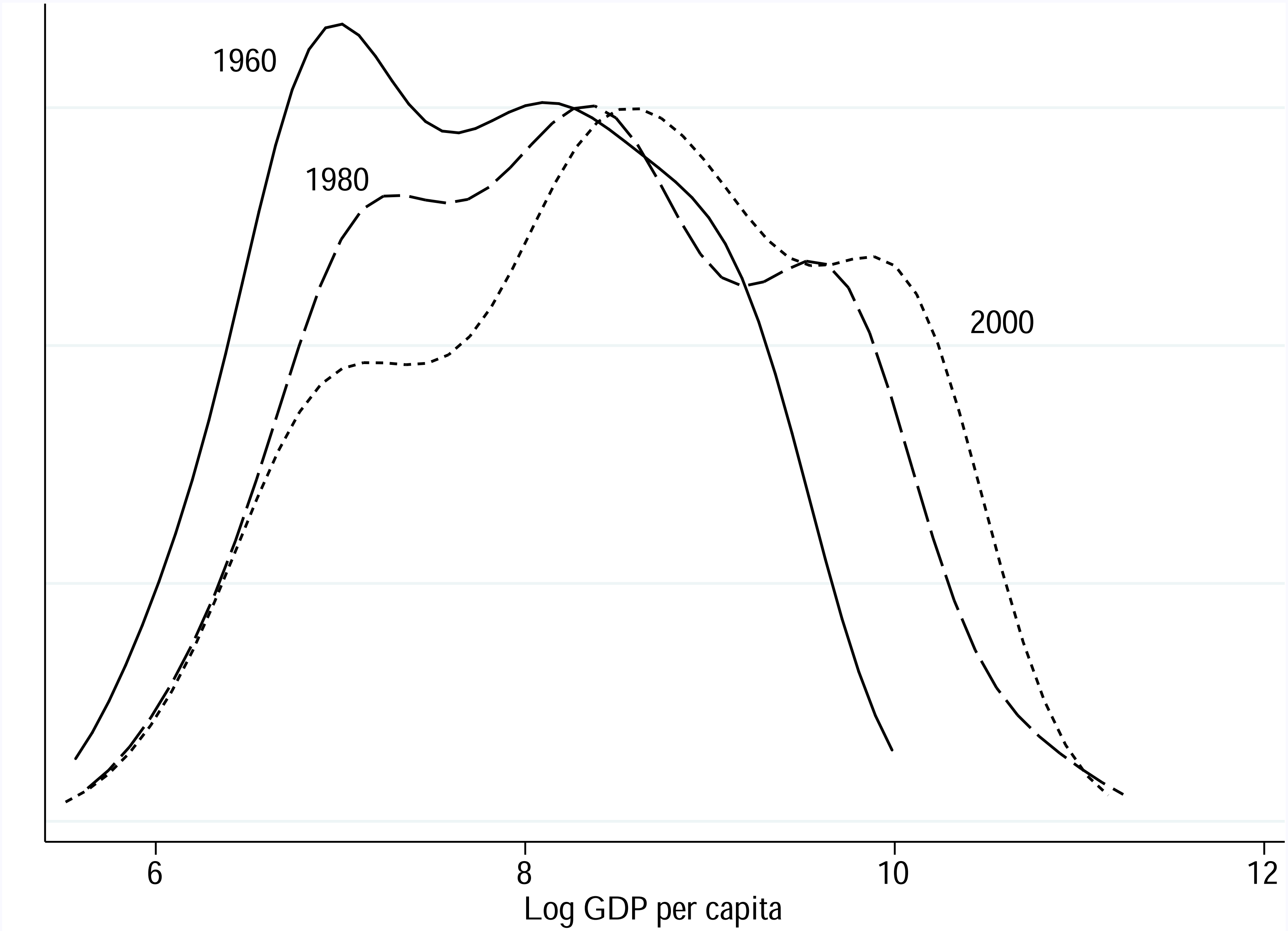


FIGURE 1.2 Estimates of the distribution of countries according to log GDP per capita (PPP adjusted) in 1960, 1980, and 2000.

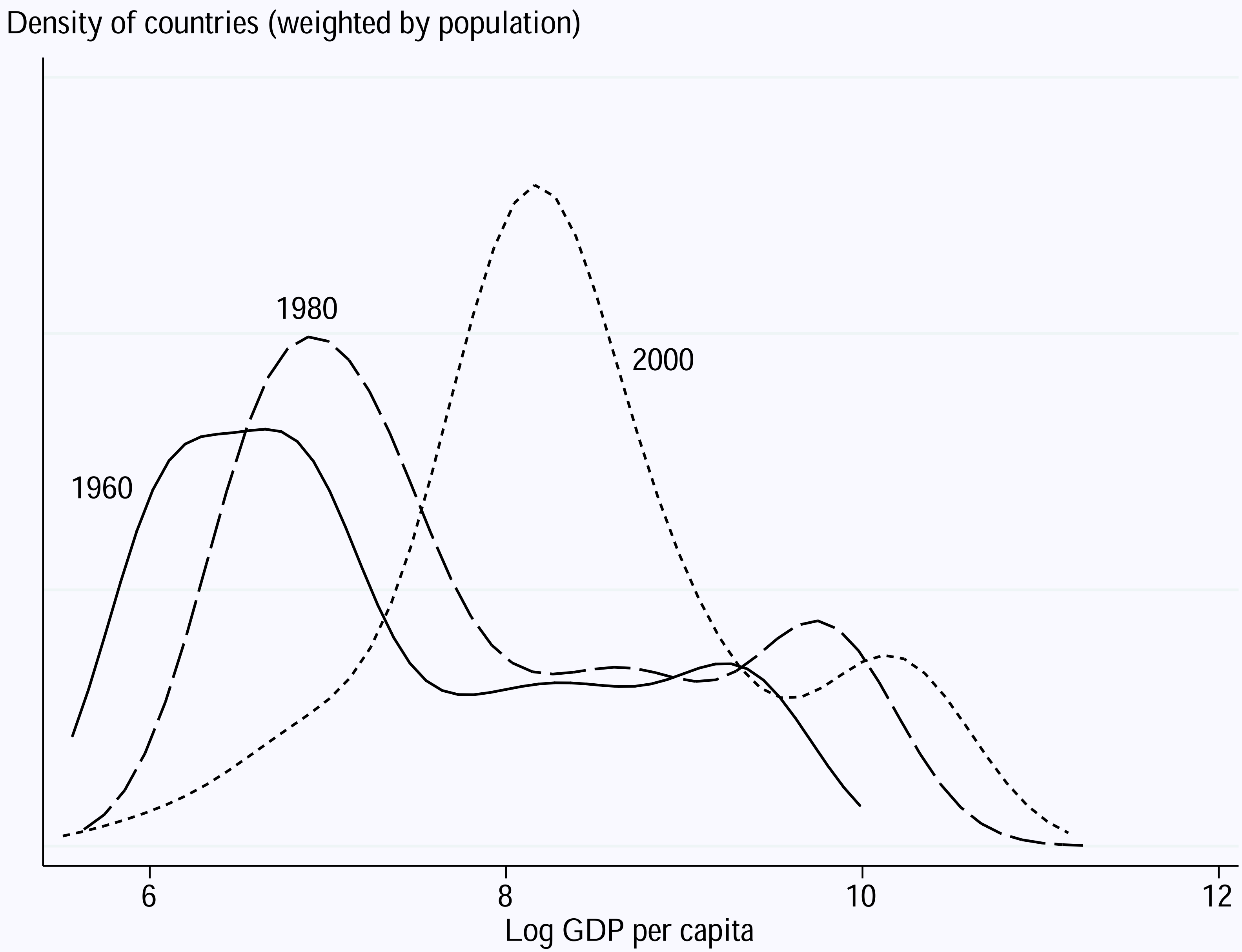


FIGURE 1.3 Estimates of the population-weighted distribution of countries according to log GDP per capita (PPP adjusted) in 1960, 1980, and 2000.

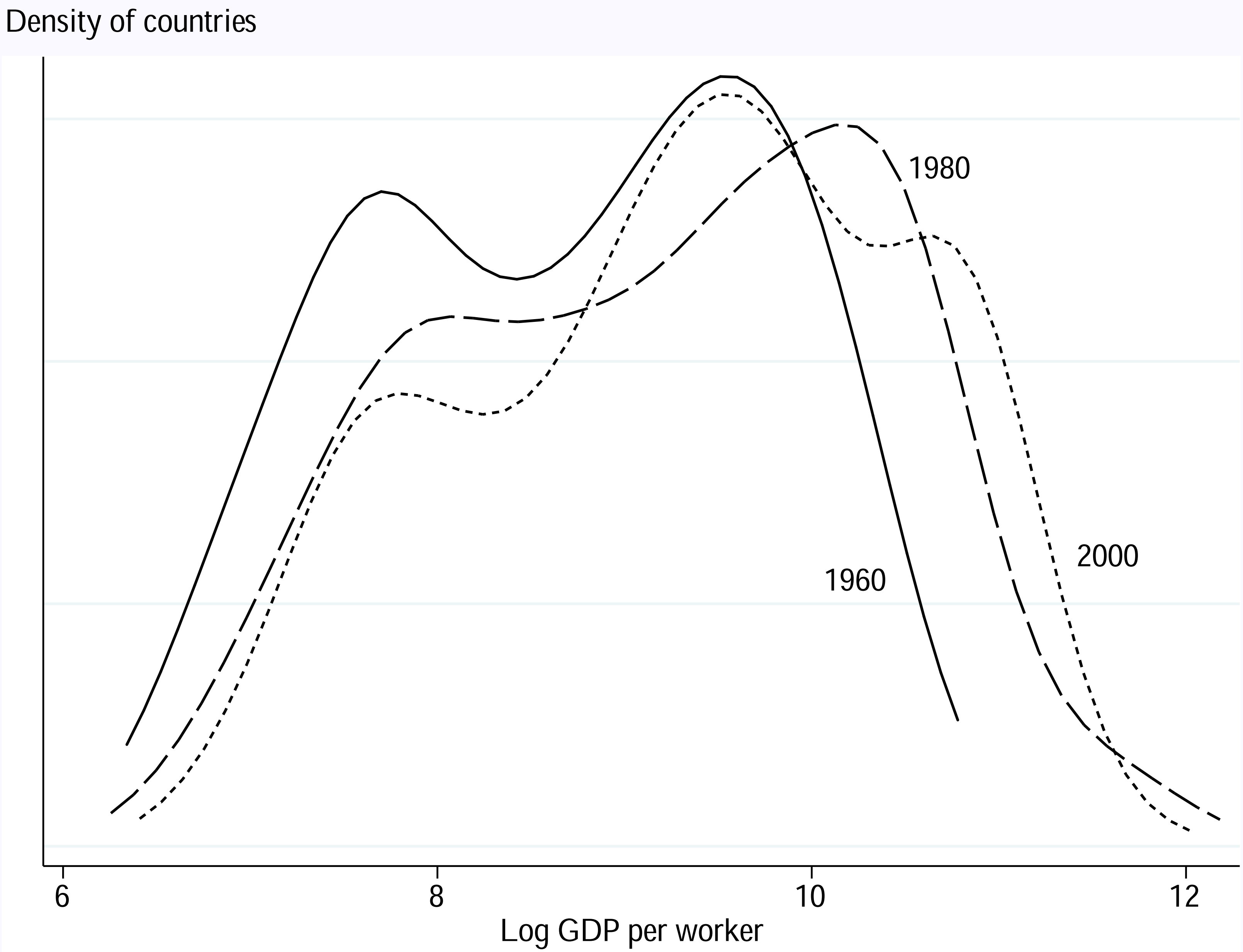


FIGURE 1.4 Estimates of the distribution of countries according to log GDP per worker (PPP adjusted) in 1960, 1980, and 2000.

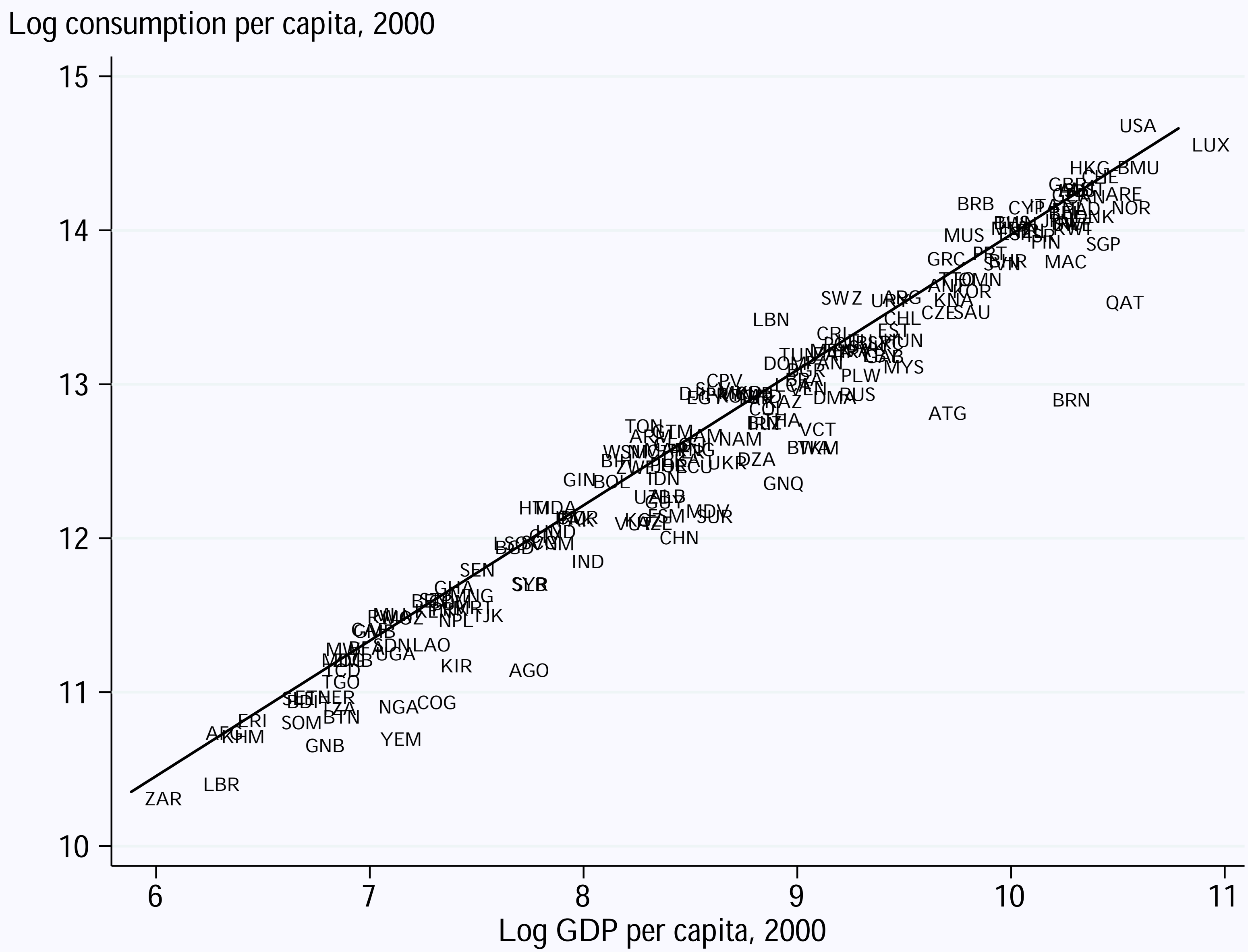


FIGURE 1.5 The association between income per capita and consumption per capita in 2000. For a definition of the abbreviations used in this and similar figures in the book, see http://unstats.un.org/unsd/methods/m49/m49alpha.htm.

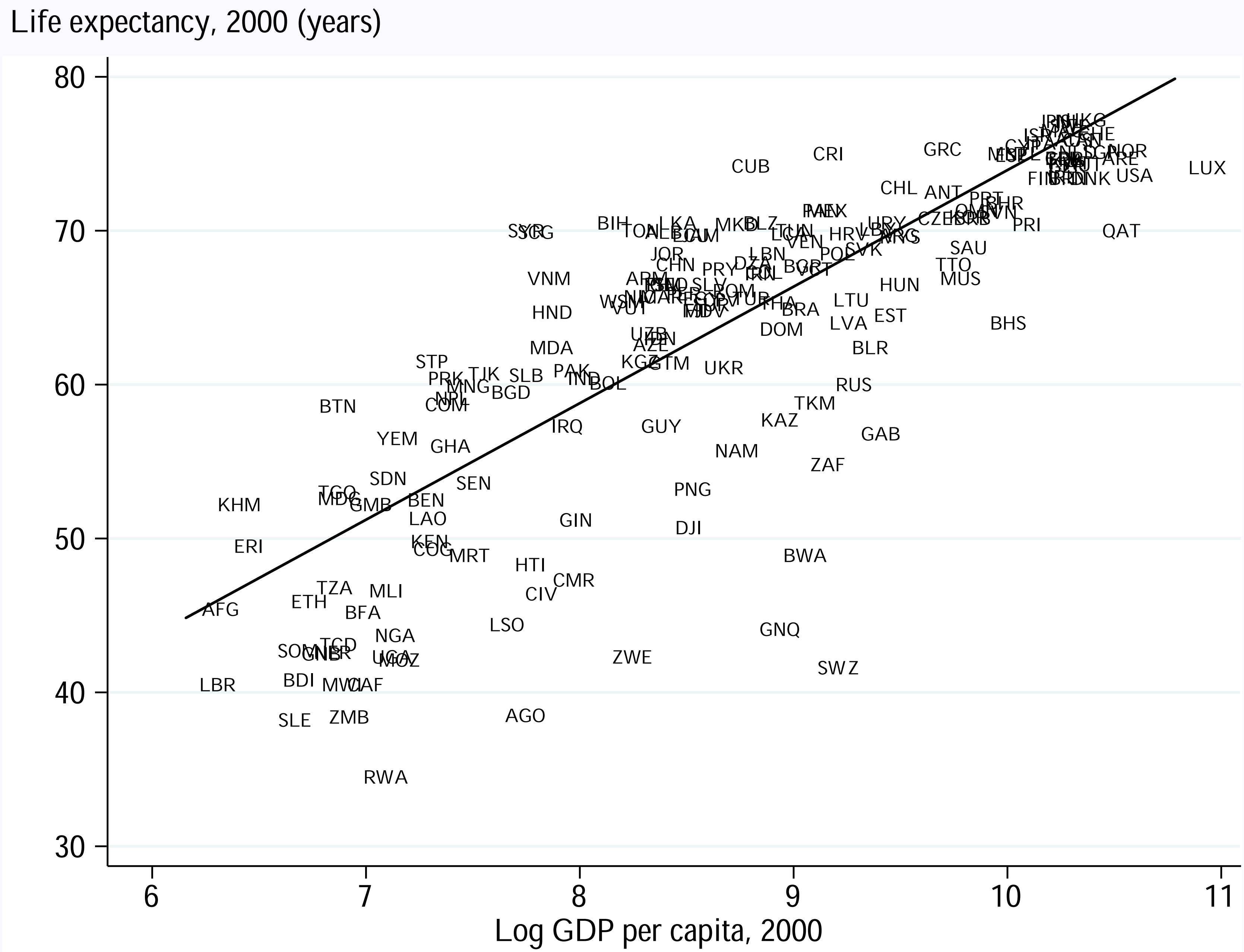


FIGURE 1.6 The association between income per capita and life expectancy at birth in 2000.

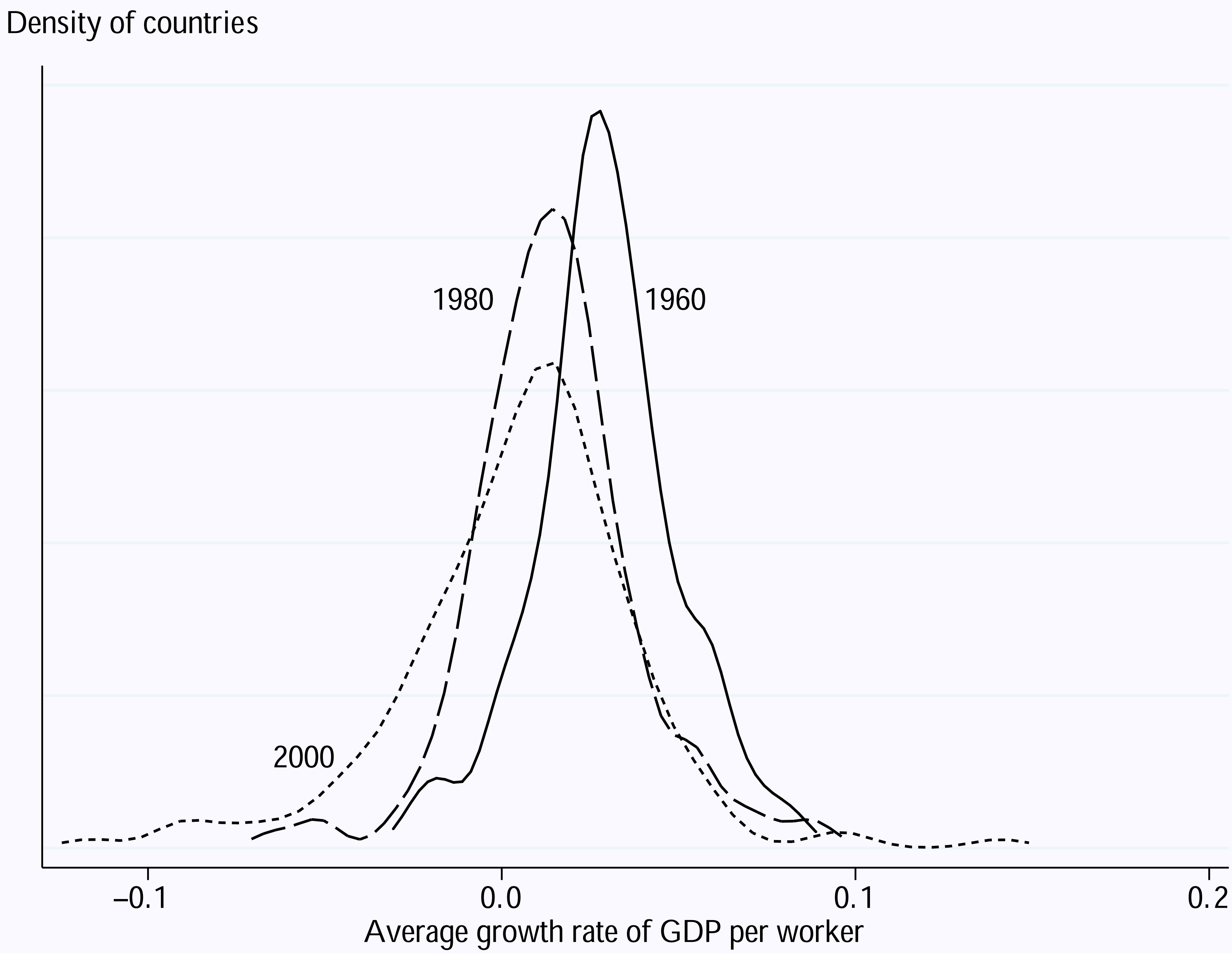


FIGURE 1.7 Estimates of the distribution of countries according to the growth rate of GDP per worker (PPP adjusted) in 1960, 1980, and 2000.

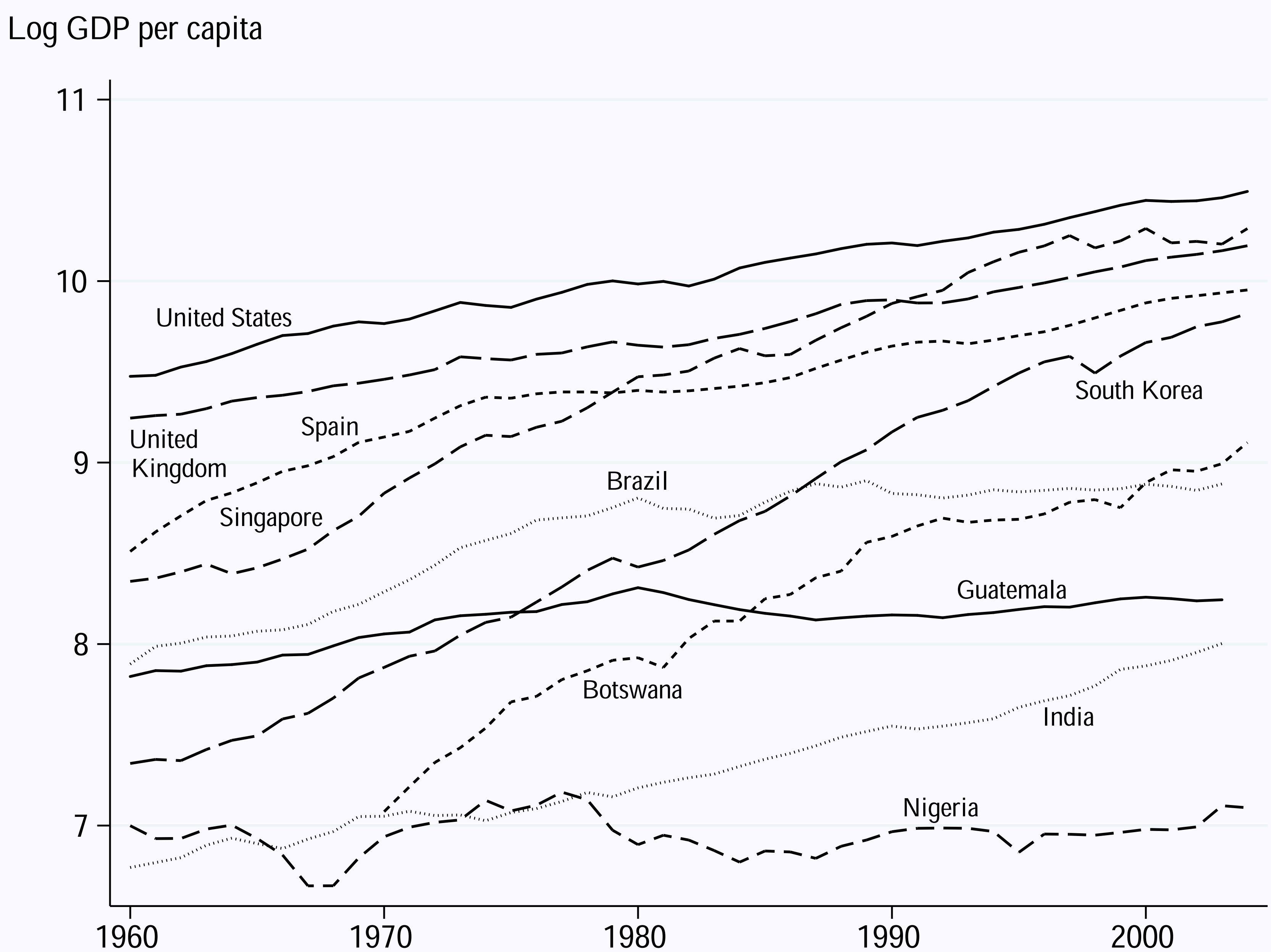


FIGURE 1.8 The evolution of income per capita in the United States, the United Kingdom, Spain, Singapore, Brazil, Guatemala, South Korea, Botswana, Nigeria, and India, 1960–2000

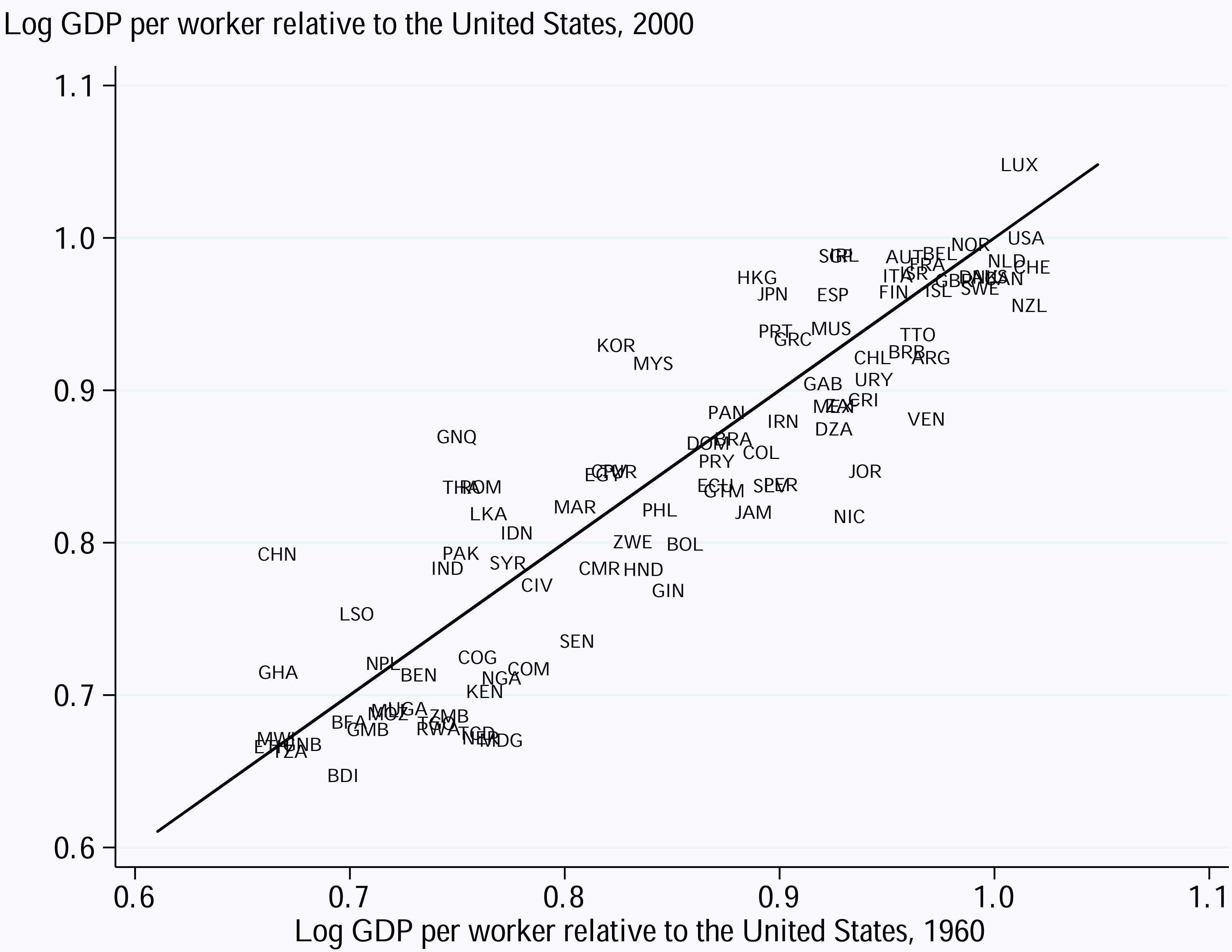


FIGURE 1.9 Log GDP per worker in 2000 versus log GDP per worker in 1960, together with the 45 line.

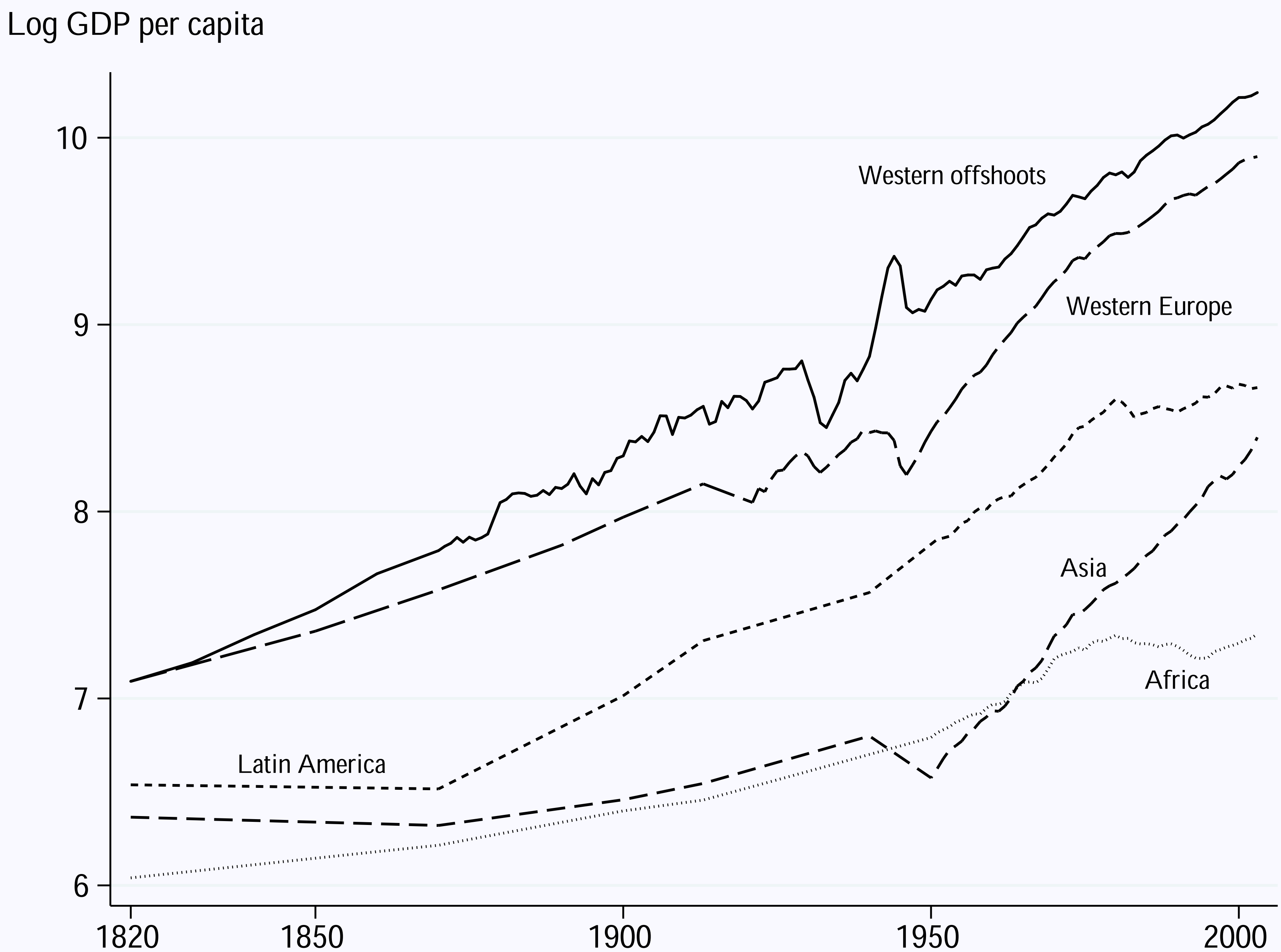


FIGURE 1.10 The evolution of average GDP per capita in Western offshoots, Western Europe, Latin America, Asia, and Africa, 1820–2000.

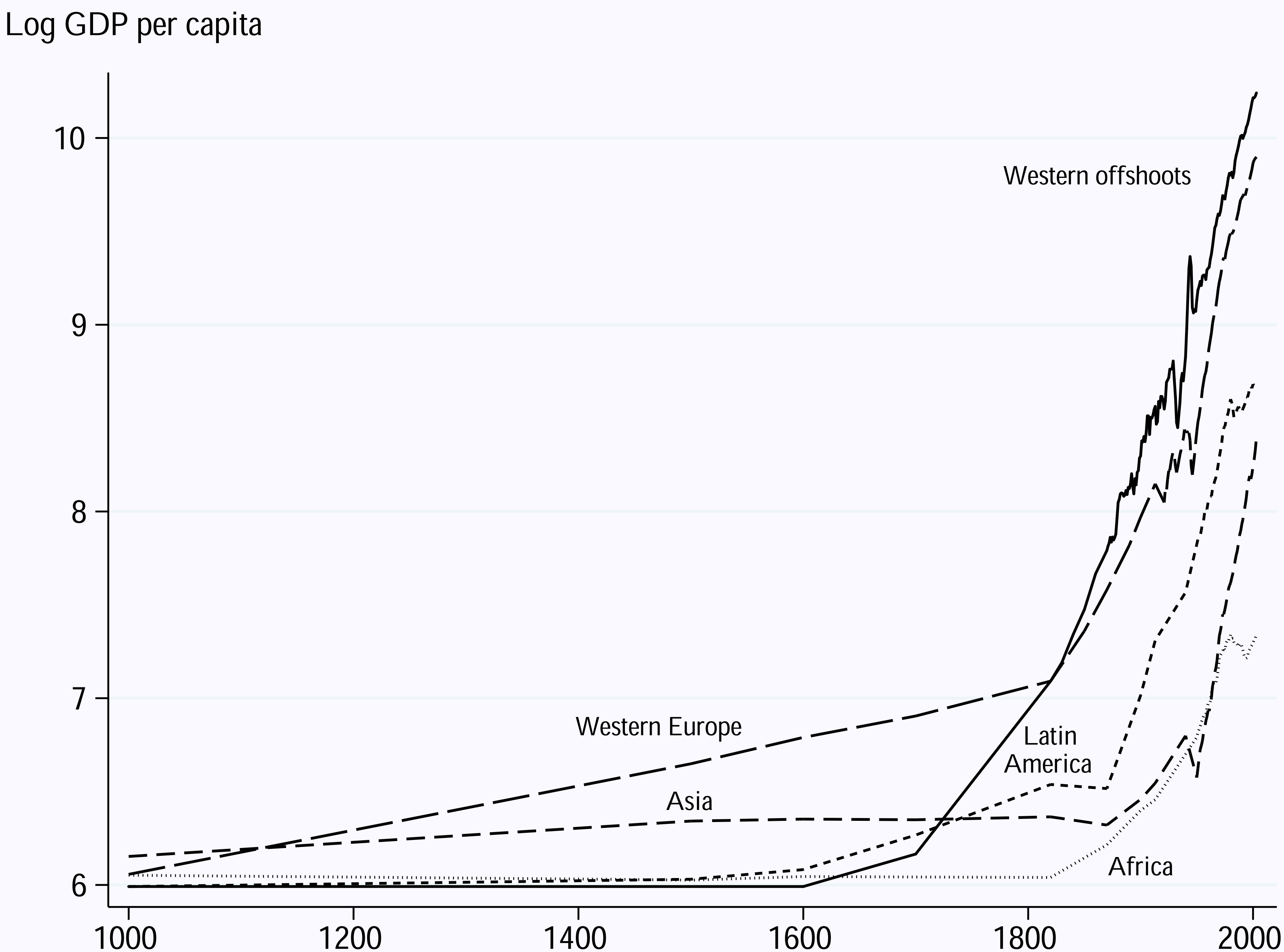


FIGURE 1.11 The evolution of average GDP per capita in Western offshoots, Western Europe, Latin America, Asia, and Africa, 1000–2000.

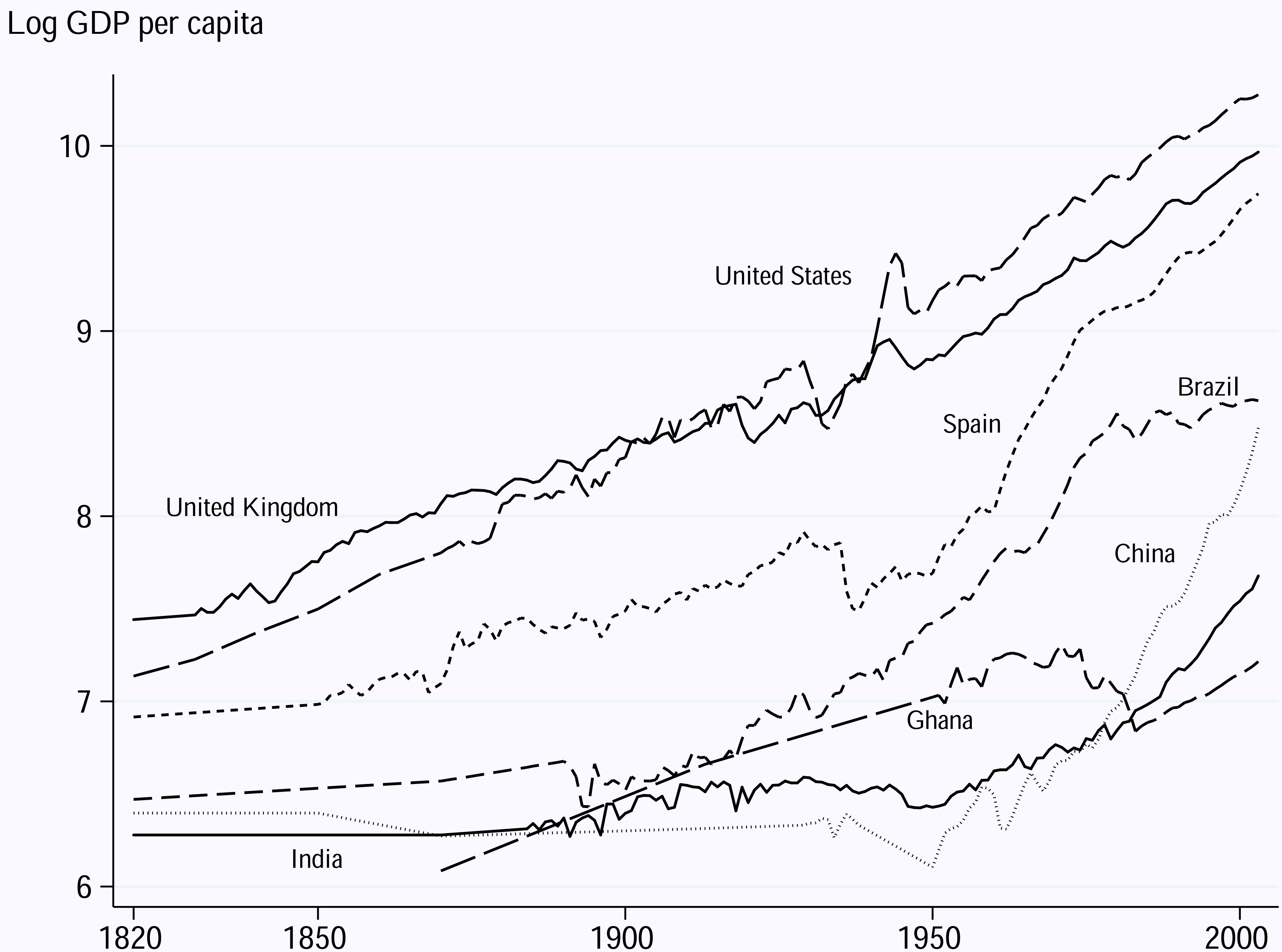


FIGURE 1.12 The evolution of income per capita in the United States, the United Kindgom, Spain, Brazil, China, India, and Ghana, 1820–2000.

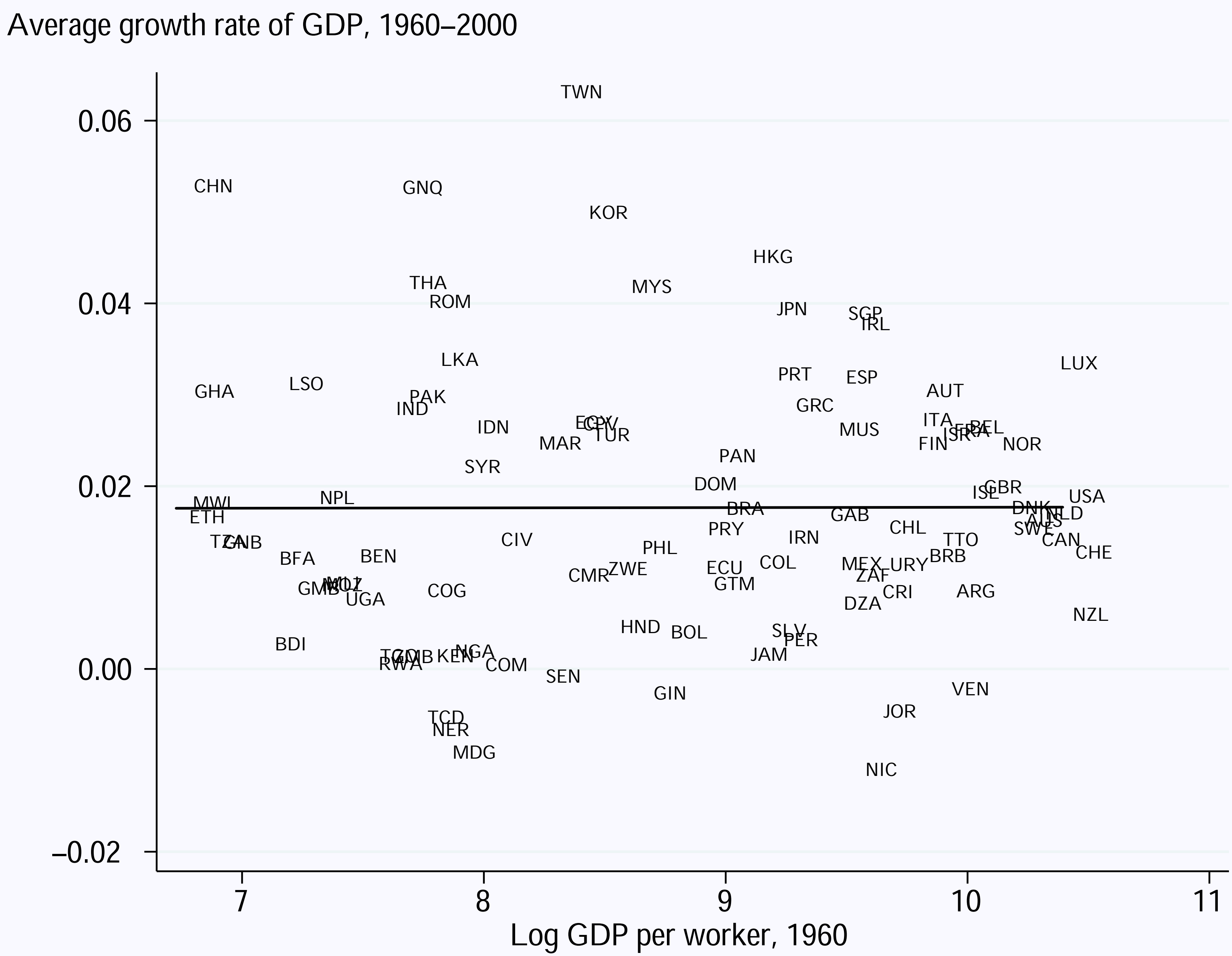


FIGURE 1.13 Annual growth rate of GDP per worker between 1960 and 2000 versus log GDP per worker in 1960 for the entire world.

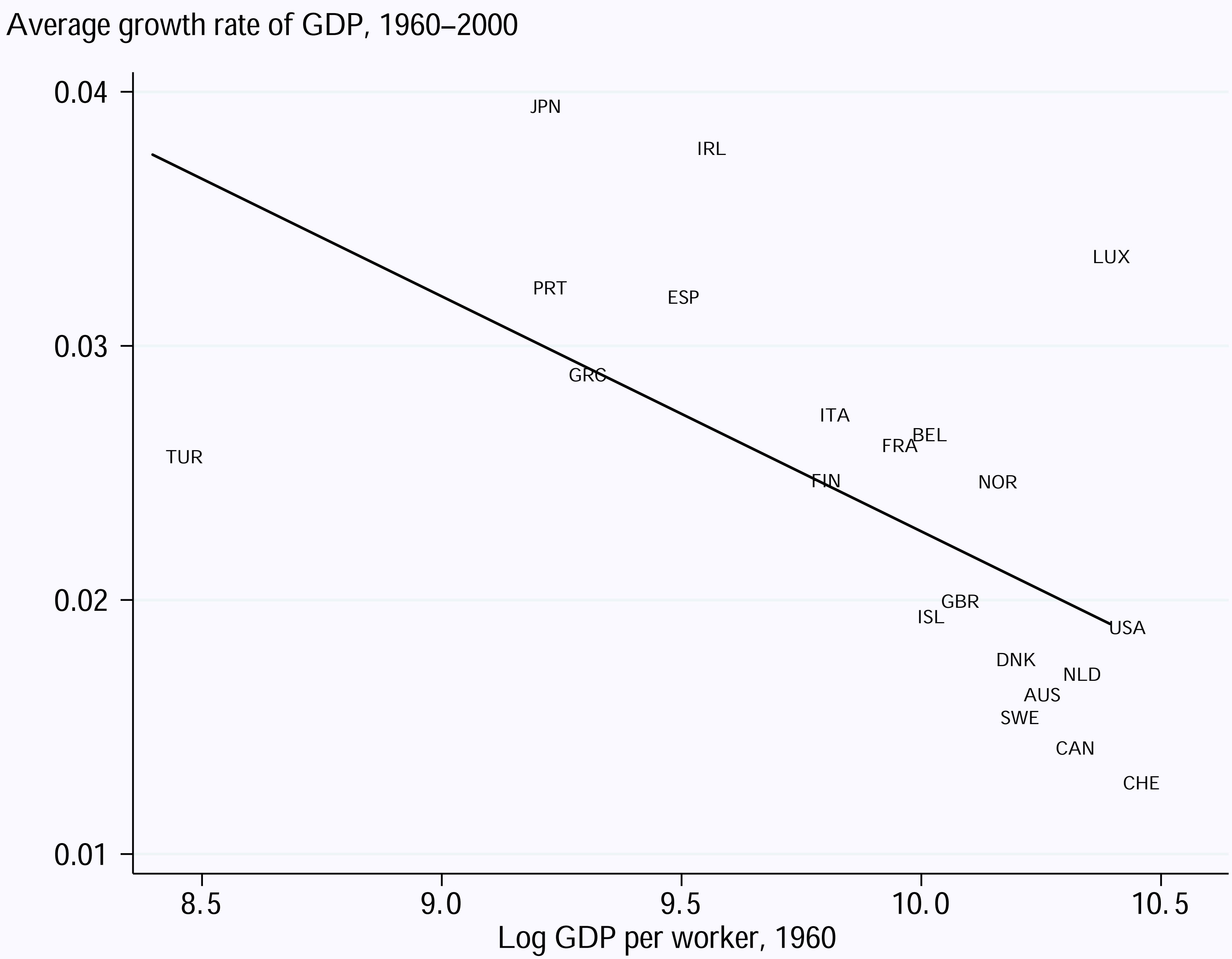


FIGURE 1.14 Annual growth rate of GDP per worker between 1960 and 2000 versus log GDP per worker in 1960 for core OECD countries.

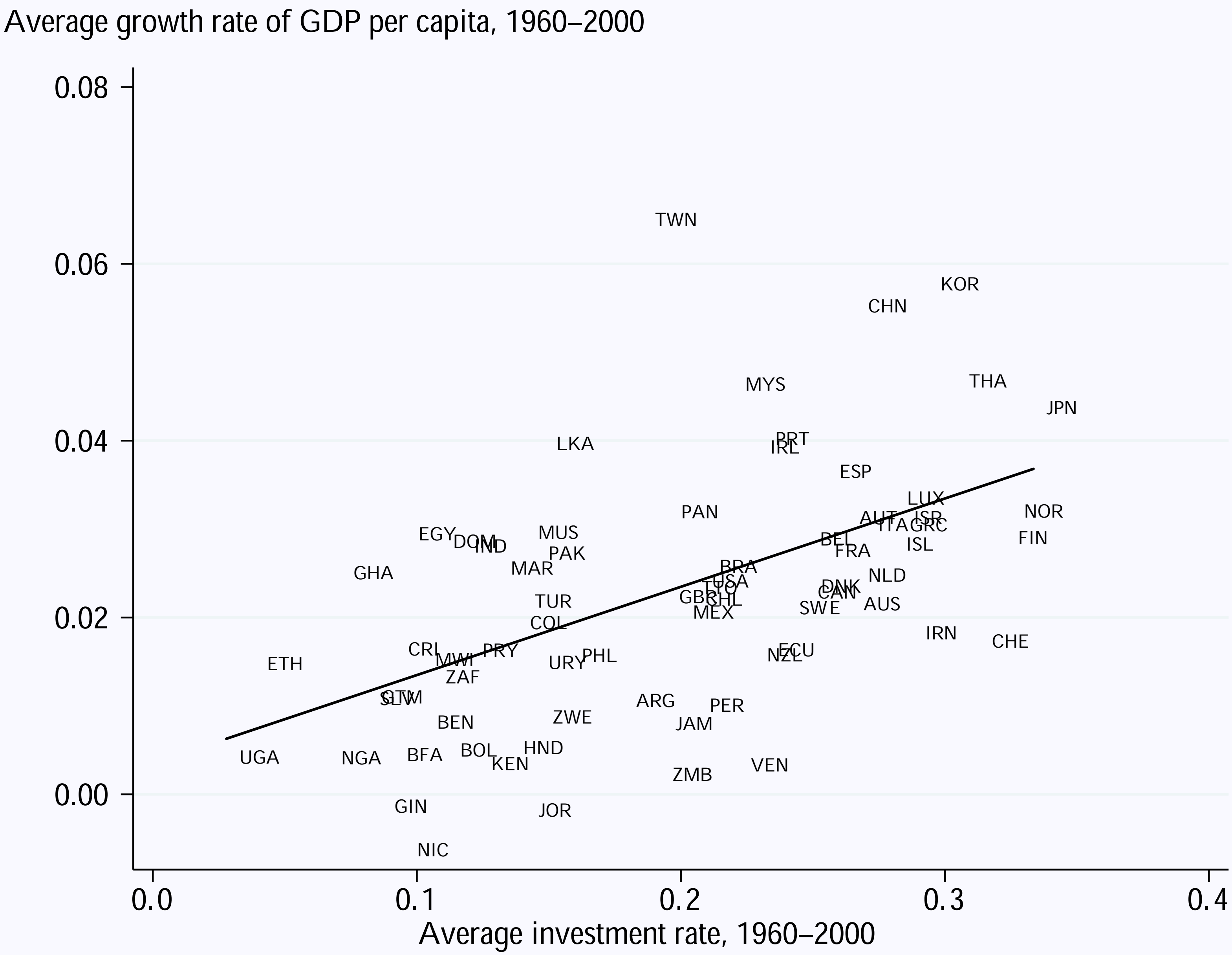


FIGURE 1.15 The relationship between average growth of GDP per capita and average growth of investments to GDP ratio, 1960–2000.

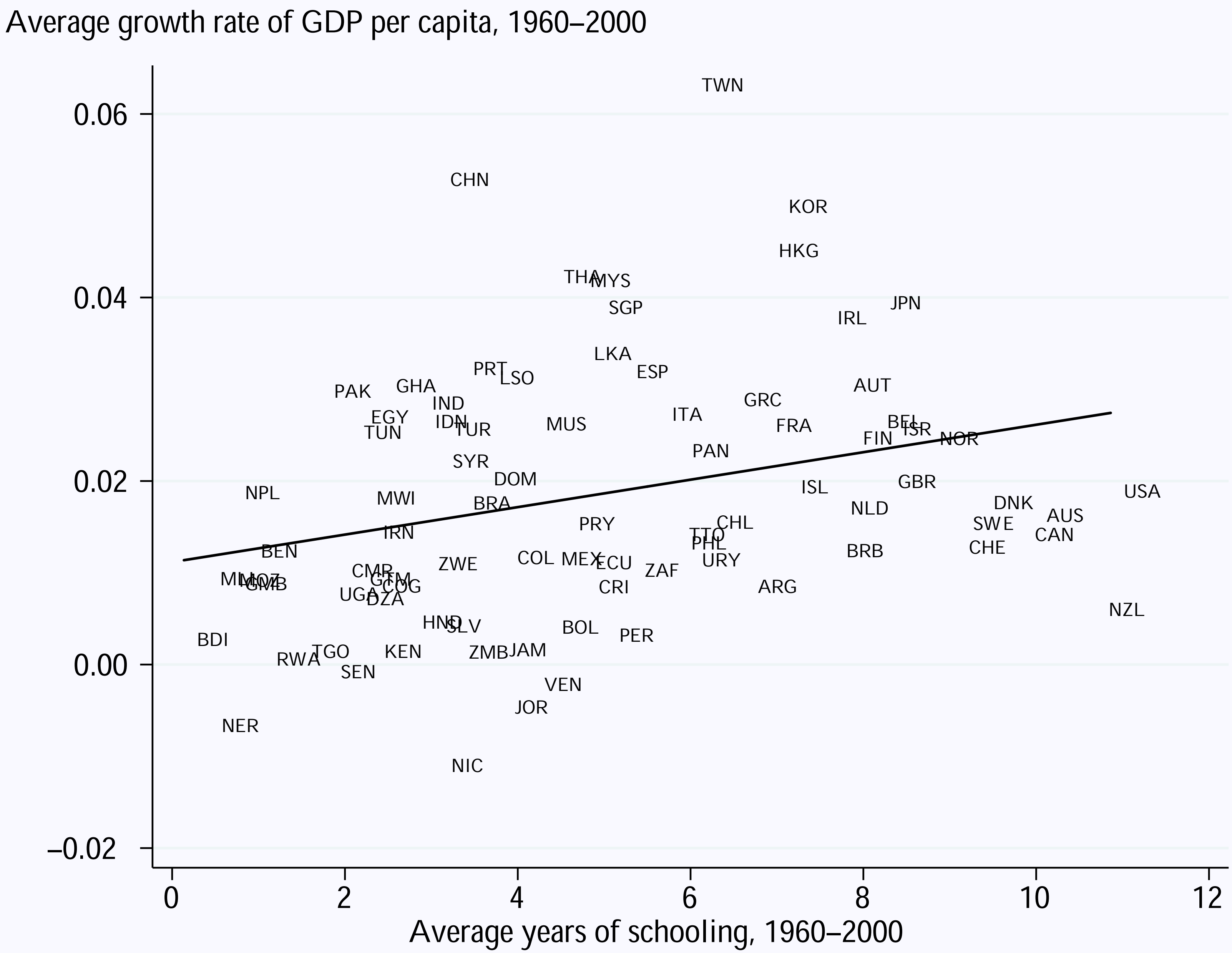
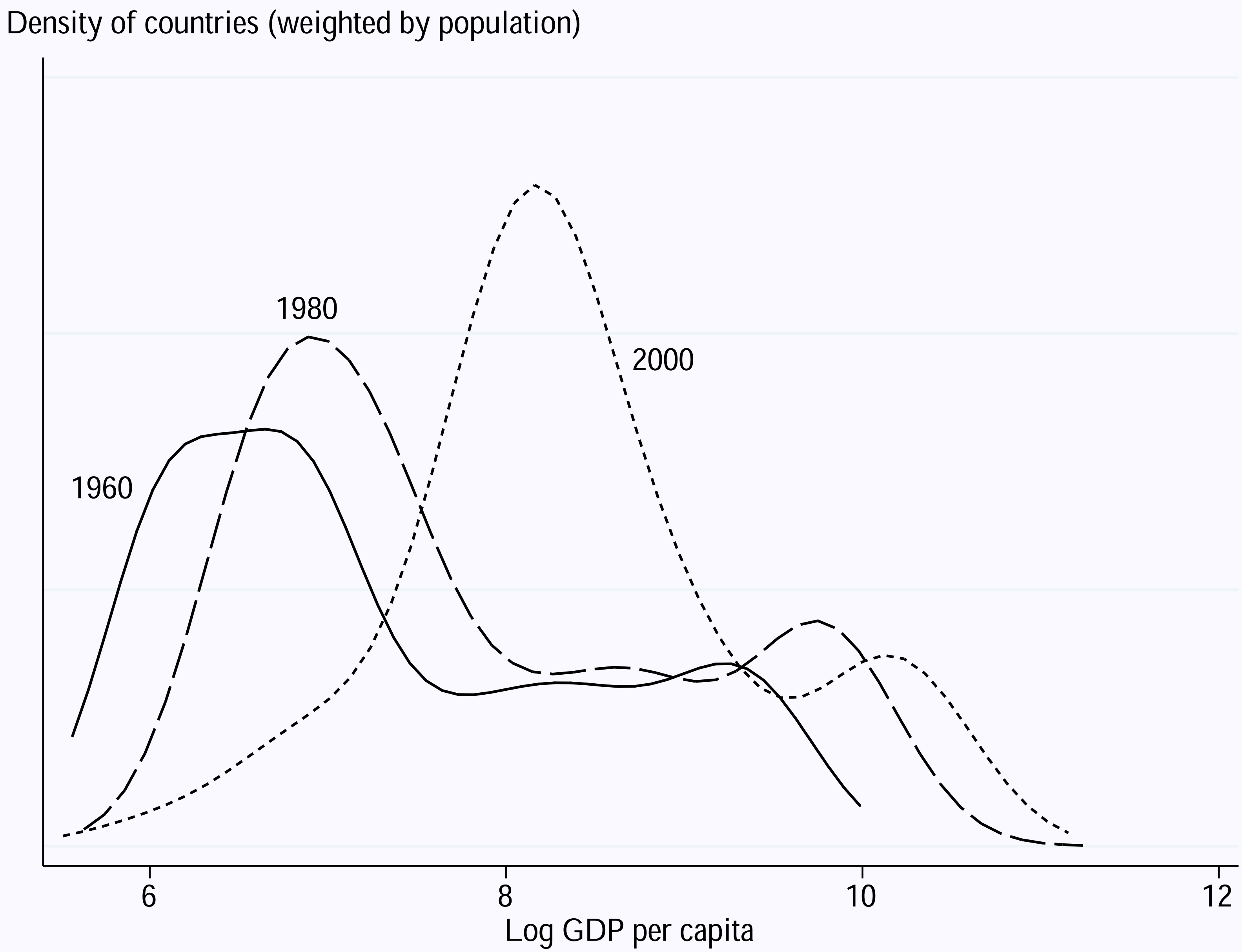


FIGURE 1.16 The relationship between average growth of GDP per capita and average years of schooling, 1960–2000.



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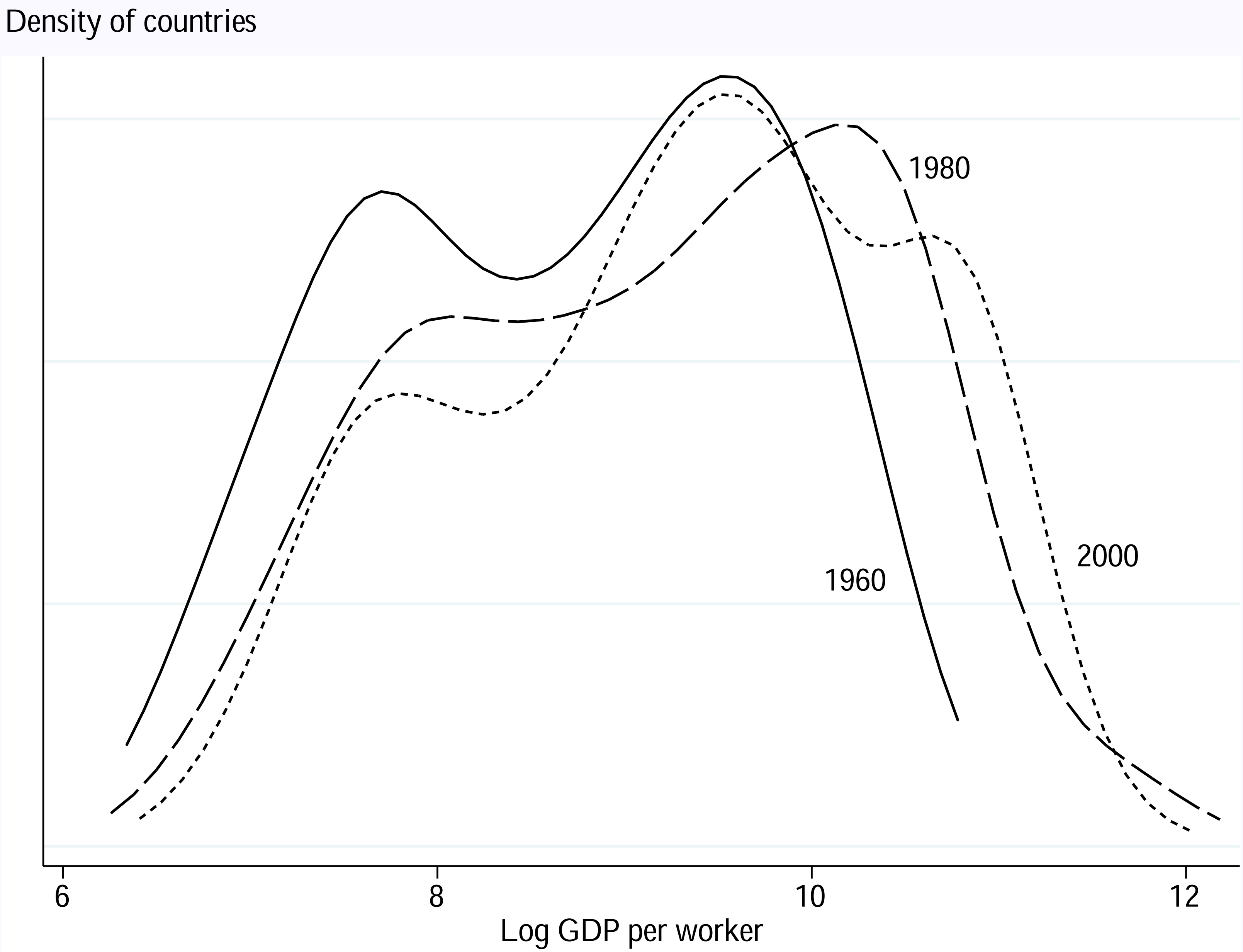

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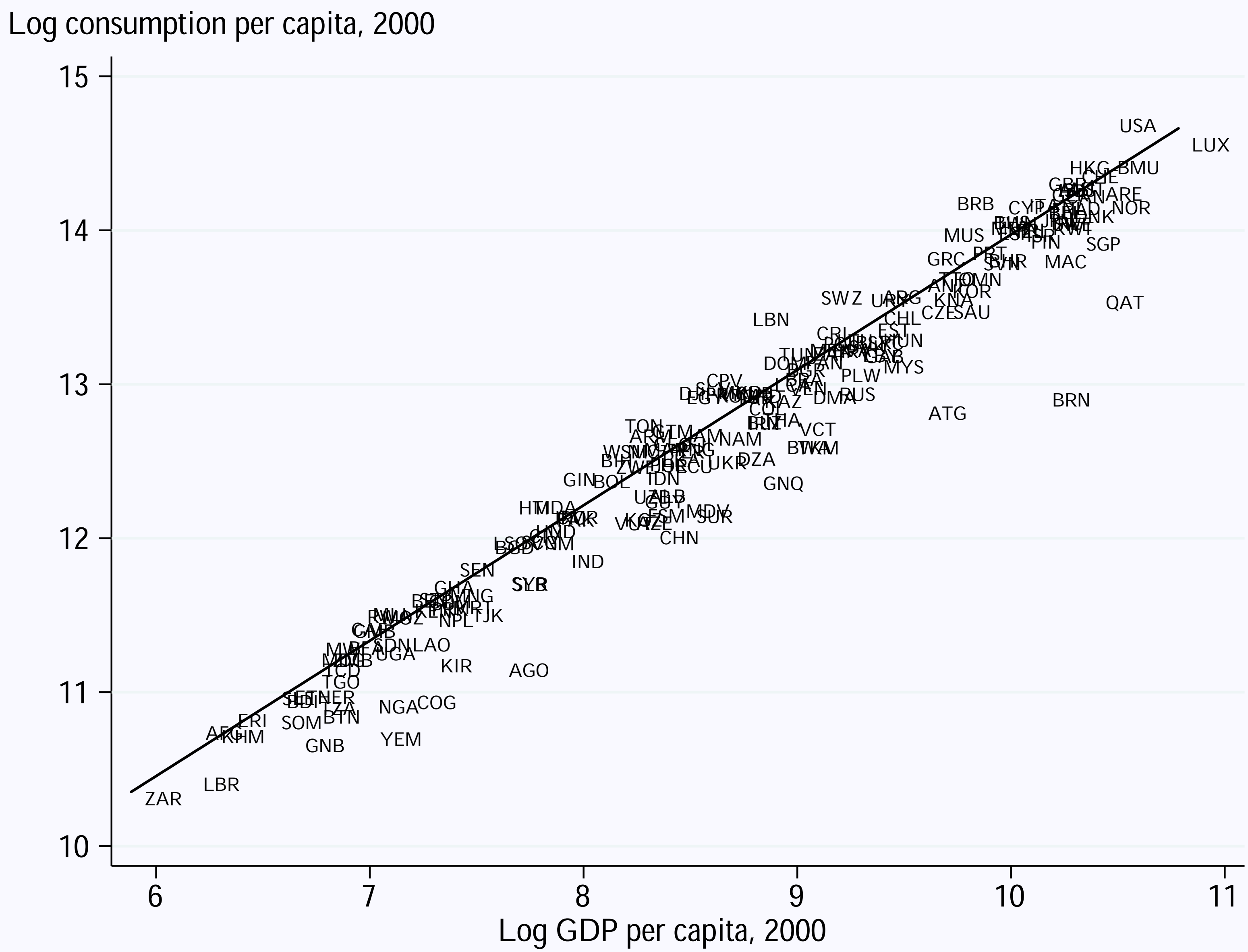

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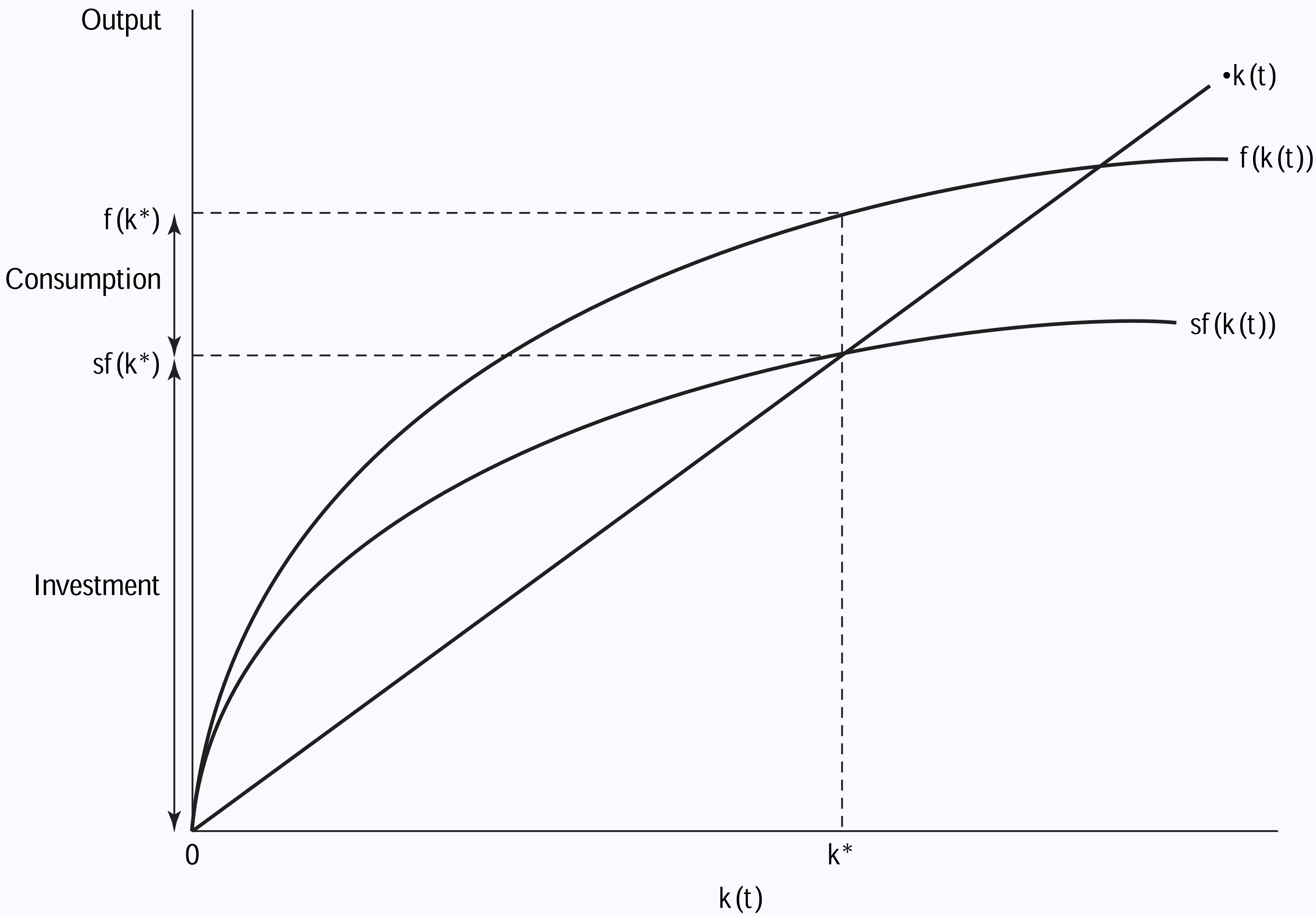


FIGURE 2.4 Investment and consumption in the steady-state equilibrium.

Long-run Growth:

Point of Departure from the short run IS-LM analysis you have may have done…

- Ignore the Demand Side
  - Assumption: Prices are flexible
  - Assumption: Agents form correct expectations
- Carefully specify the supply side
  - L Labour is exogenously given
  - K Capital is endogenous over time
  - A Technology exogenous to start with …

THE PRODUCTION FUNCTION

$$Y = F(K, L) \quad \text{where} \quad \begin{aligned} Y &= \text{output} \\ K &= \text{capital (input / factor)} \\ L &= \text{labour (input / factor)} \end{aligned}$$

- Assumptions:
  - *Constant Returns to Scale:* scale up or scale down the economy – on worker firm can gives us the intuition of the economy
  - *Marginal Products positive and diminishing:* factors are productive, but at a decreasing rate.

Marginal Products

- Marginal Product of Labour:

$$\begin{aligned} \frac{\partial Y}{\partial L} &= F_L && > 0 && \textit{positive} \\ \frac{\partial^2 Y}{\partial L^2} &= F_{LL} && < 0 && \textit{and diminishing} \end{aligned}$$

  - labour contributes positively to the output but each subsequent contribution is smaller.

- Marginal Product of Capital:

$$\begin{aligned} \frac{\partial Y}{\partial K} &= F_K && > 0 && \textit{positive} \\ \frac{\partial^2 Y}{\partial K^2} &= F_{KK} && < 0 && \textit{and diminishing} \end{aligned}$$

  - capital contributes positively to the output but each subsequent contribution is smaller.

Constant Return to Scale → Representative Firm

- Size does not matter
  - The whole country could be one firm
  - Alternative, the country could be divided into infinite number of tiny firms
- We try to understand the economy by understanding the action of a single *representative* firm

Cobb-Douglas Production Function

$$Y = K^{\alpha}(AL)^{1-\alpha}$$

- A: constant (represents state of technology)
- A plays a key role in theory of growth
- check whether Cobb-Douglas Production function ...
  - exhibits constant returns of scale?
  - has complementarity of factors?
  - satisfies the Euler's theorem?

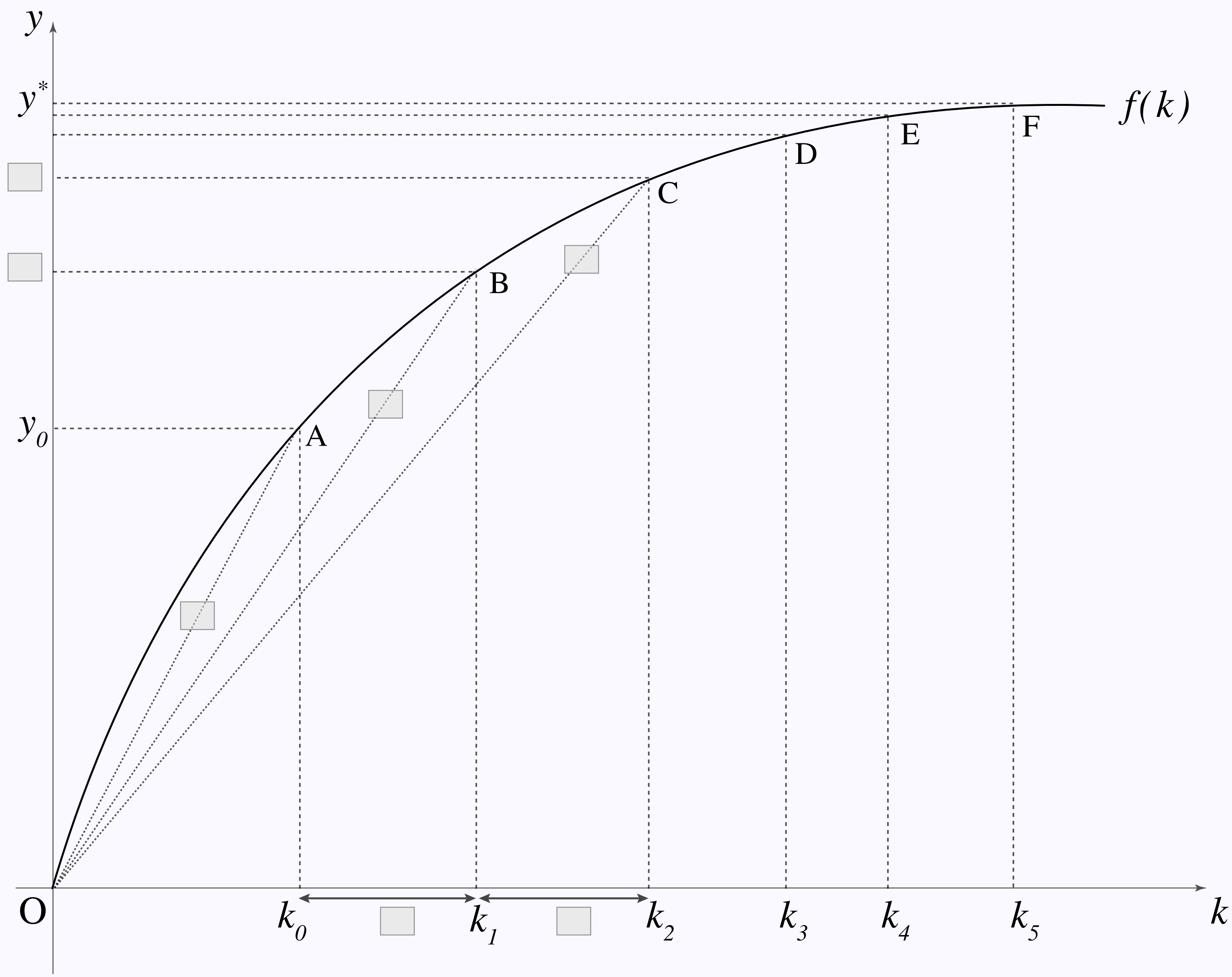


Figure: Per worker production function

Output capita and Capital Output Ratio

Lets take a take a constant returns to scale production function where output per capita is a function of capital per capita

$$y = f(k) \tag{1}$$

- By looking at the Figure *per worker production function*, convince yourself that the output per worker (y) increase concomitantly with capital stock per worker (k)
- Is there a clear relationship between growth of capital stock per worker k and growth of output per capital stock y?
- Is there a limit to growth of k and y?
- What happens to the output per capital stock $\left(\frac{y}{k}\right)$ as capital stock per worker (k) increases?

If we can understand the growth dynamics for the above production function, we can easily use different kinds of production functions to understand the growth process of the economy. E.g.

$$Y = AK^{\alpha}L^{1-\alpha}$$

$$Y = (AK)^{\alpha}A^{1-\alpha}$$

We can even add other factors of production like H, human capital.

$$Y = AK^{\alpha}H^{\beta}(L)^{1-\alpha-\beta}$$

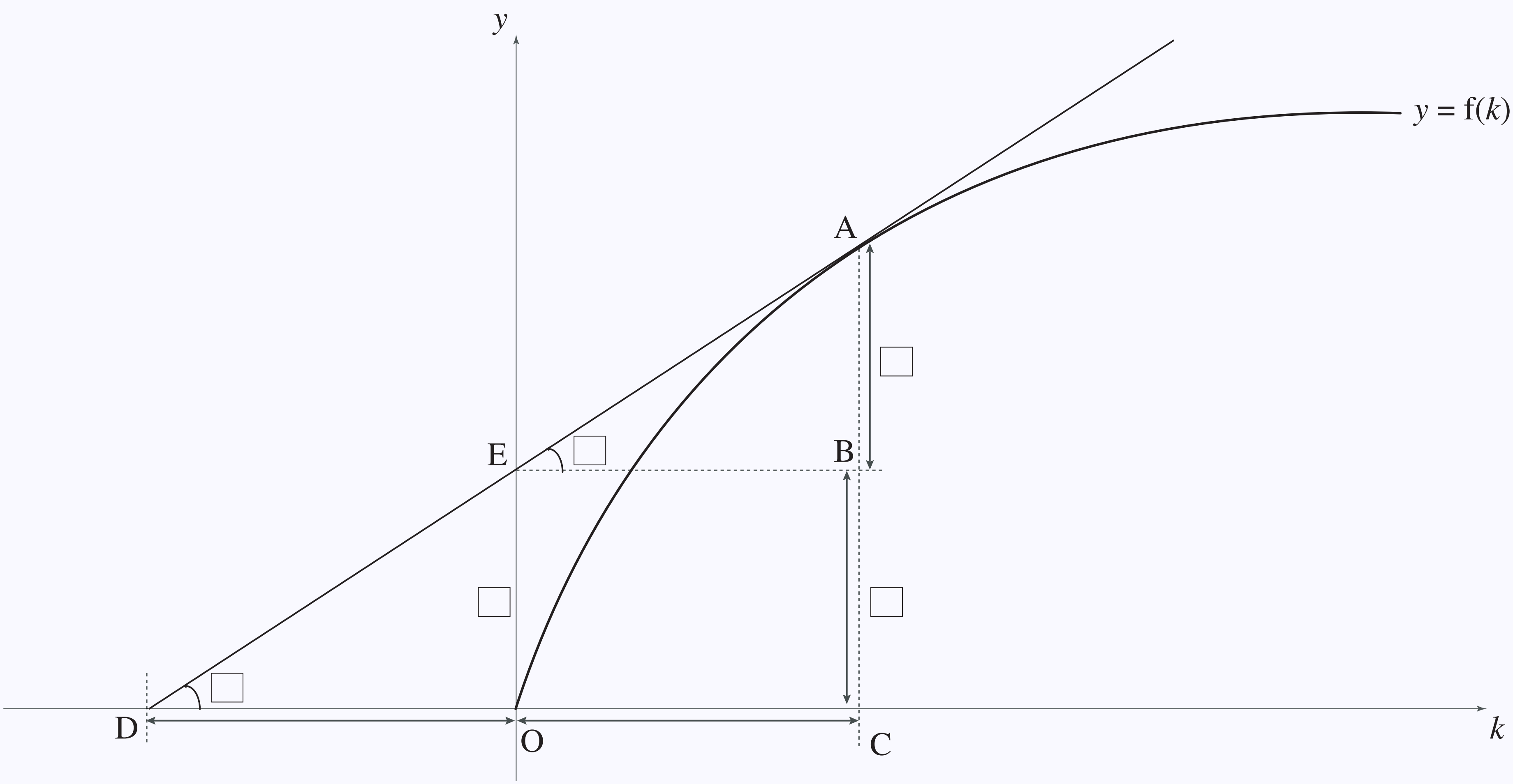


Figure: Constant Returns to Scale Production Function

Facts of Growth: The Kaldor Facts

- $\frac{K}{L}$ grows at constant rate
- r is constant
- $\frac{Y}{K}$ is constant
- w grows at a constant rate
- Growth rate & Income levels vary substantially across countries
- Growth rate not necessarily constant over time

BEHAVIOURAL FUNCTIONS

Saving: people save a constant (s) proportion of their income

$$S = s \cdot Y$$

Consumption: people consume $(1 - s)$ proportion of their income

$$C = (1 - s) \cdot Y$$

Investment: all the saving in the economy gets invested

$$S = I$$

Depreciation

- Depreciation: Capital replaced due to wear and tear. *machinery needs to be serviced in order to be brought back to its original condition*
- Capital depreciates at the rate of δ
- Capital Formation: *Any addition to capital stock first gets absorbed by depreciation and the residual gets added to capital stock.*

Two kinds of Investment

- Replacement Investment: compensates for depreciation δK amount of capital depreciates & has to be replaced every period
- Net Investment: brand new capital stock or new machinery added to the economy in a period

Capital Formation

- Today's investment is tomorrow's capital

$$
\begin{aligned}
I &= K_{t+1} - K_t + \delta K_t \\
&= \Delta K_t + \delta K_t
\end{aligned}
$$

Investment today is	*compensation for depreciation:*	δK_t
	addition to capital Stock:	$\Delta K_t = K_{t+1} - K_t$

The Fundamental Equation

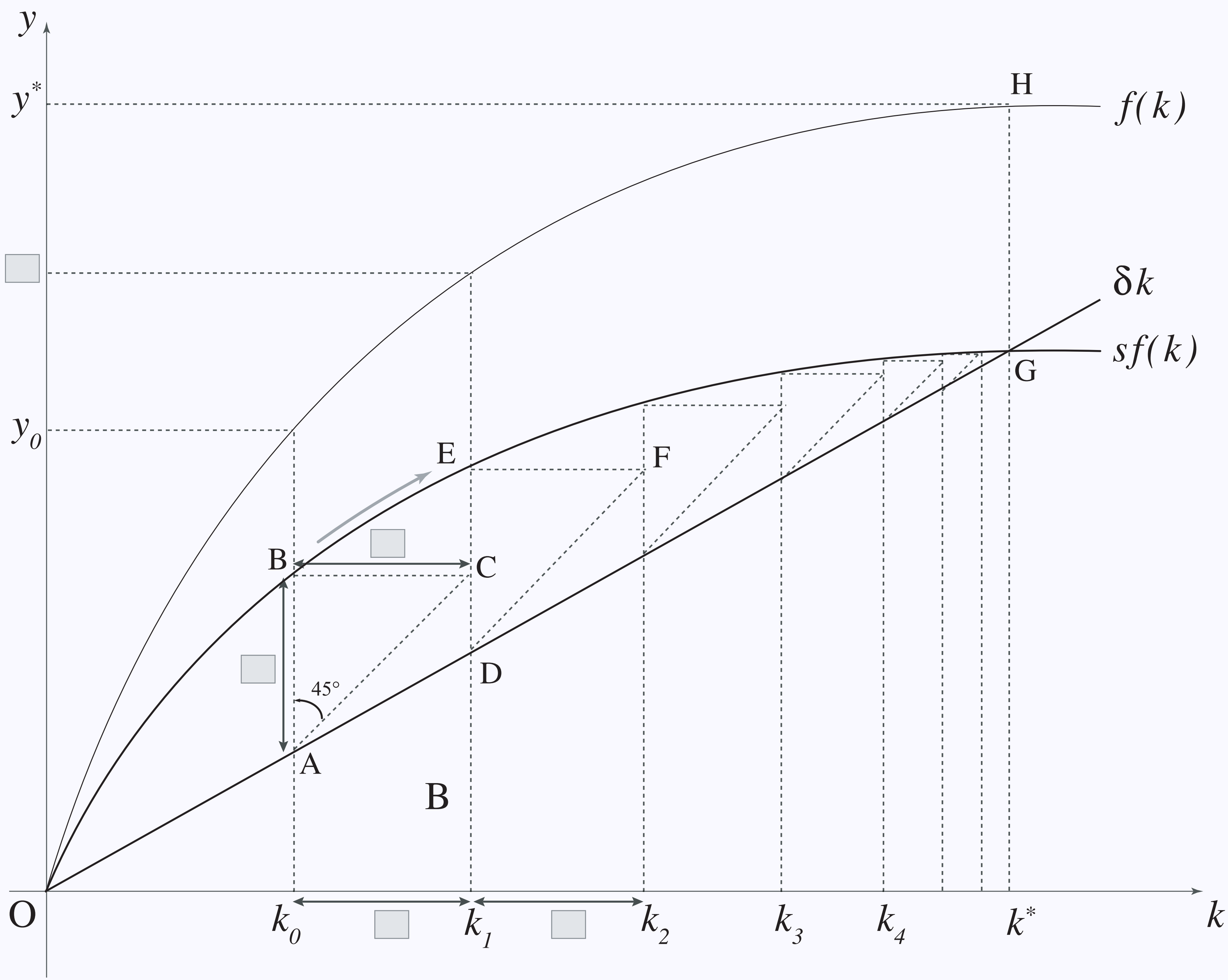
Saving Investment Equality

$$S = I$$

$$sY = \Delta K_t + \delta K_t$$

Fundamental equation:

$$\frac{\Delta k_t}{k_t} = s\left[\frac{y_t}{k_t}\right] - \delta$$


y
y*
H
f(k)
δk
sf(k)
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y0
E
F
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45°
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A
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k0
k1
k2
k3
k4
k*
k

- In the Figure: *Growth of capital per capita*, start from an arbitrary capital stock per worker k_0 and find the net increase in k per period
- Illustrate how *capital stock per worker* increases from k_0 to k^* and *output per worker* increases from y_0 to y^*.
- Why does *capital stock per worker* not increase beyond k^*?
- Do you notice a pattern in the rate at which capital stock per worker (k) and output per worker (y) grows?
- Draw yourself another diagram and show what would happen if we start from a situation where *capital stock per worker* of the economy is greater than k^* to start with. per worker (k) grows?

The Solow Model

Definition (Solow Model I)

The most basic Solow model with no population growth or technological progress.

Assumption

a) *no population growth* $\Rightarrow \frac{\Delta L}{L} = 0$

b) *no technological progress* $\Rightarrow \frac{\Delta A}{A} = 0$

Fundamental Equation - I

Definition (Fundamental Equation - I)

$$\Delta K_t = s \cdot Y_t - \delta \cdot K_t$$

the part of savings that does not get absorbed by depreciation gets added to capital stock

- *the fundamental equation*
  - *derived from the saving investment equality*
  - *would change to account for population growth*
  - *would change to account for technological progress*

Growth Rate of Capital Stock

$$\Delta K_t = sY - \delta K_t$$

$$\frac{\Delta K_t}{K_t} = s\frac{Y_t}{K_t} - \delta$$

- growth rate of capital
  - increases with the saving rate s
  - decrease with rate of depreciation δ
  - increases with output-capital ratio $\frac{Y_t}{K_t}$
    - $\frac{Y_t}{K_t}$ decreases as K_t increases

Steady State

Definition (Steady State)

The economy reaches the steady state when the endogenous variable stop changing

In Solow Model - I

- Endogenous variable: K_t
- Steady State Condition: $\frac{\Delta K_t}{K_t} = 0$

$$\frac{\Delta K_t}{K_t} = s\frac{Y_t}{K_t} - \delta = 0$$

$$\Rightarrow \left[\frac{Y_t^*}{K_t^*}\right] = \frac{\delta}{s}$$

Growth in Steady State

- In Steady State

$$\left[\frac{Y_t^*}{K_t^*}\right] = \frac{\delta}{s}$$

  - output-capital ratio is constant
  - Capital stops growing
  - Output Stops growing
  - No growth in steady state

- Does this match our observation of the world?

Convergence Dynamics

Definition (Convergence Dynamics)

The evolution of the endogenous variables as the economy moves toward the steady state

- When the economy is not in steady state

$$\frac{\Delta K_t}{K_t} = s\frac{Y_t}{K_t} - \delta > 0$$

$$\frac{Y_t}{K_t} > \frac{\delta}{s}$$

$$\frac{Y_t}{K_t} > \left[\frac{Y_t^*}{K_t^*}\right]$$

Convergence Dynamics

$$\frac{\Delta K_t}{K_t} = s\frac{Y_t}{K_t} - \delta \qquad = s\left(\frac{Y_t}{K_t} - \left[\frac{Y_t^*}{K_t^*}\right]\right)$$

- Assume: $K_t < K_t^*$
  - as $K_t \uparrow$, capital-output ratio $\downarrow$
  - growth rate of capital is the difference between current capital-output ratio and steady state capital-output ratio
  - the further away from steady state the economy is, the faster the rate at which capital grows
  - the further away from steady state the economy is, the faster the rate at which output grows

Summary

- Steady state is determined by s, δ and L
  - a higher $s \Rightarrow$ a higher K^* and Y^*
  - a higher $\delta \Rightarrow$ a lower K^* and Y^*
  - a higher $L \Rightarrow$ a higher K^* and Y^*
- Solow Model - I says that poor countries are poor because
  1. their depreciation rate δ is high (unlikely)
  2. their saving rates s are low (unlikely)
  3. their level of technology is low (most likely)

Summary

- Convergence dynamics are determined by "distance" to steady state
- the further the economy is from steady state
  - the faster K grows
  - the faster Y grows
- explains the take - off phase of growth
  - Germany and Japan in 30 years after World War II
  - When reform raises factor productivity i.e. China, India

Puzzle

Puzzle: According to Solow Model - I economic growth (of K and Y) can only be achieved if the economy is not in steady state. Once it reaches steady state, there is no growth.

- This obviously contradicts our observation of the world around us
- We need to enrich the model with population growth and technological progress to see if it can provide us with a better explanation.

Definition (Solow Model)

Solow model with positive population growth and technological progress.

Assumption

a) *Positive population growth* $\Rightarrow \frac{\Delta L}{L} = n > 0$

b) *Positive technological progress* $\Rightarrow \frac{\Delta A}{A} = g > 0$

Technology in Solow Growth Model

Definition (Labour-augmenting Technology)

$$Y = F(K, AL)$$

- *technological progress occurs when A increases over time*
- *a unit of labour becomes more productive with technological progress (as A increases)*

⊙ What happens to the production function as A increases?

Effective Units

Definition (Effective Units of Labour)

- *AL is defined as the efficiency units of labour.*
- *Labour augmenting technological progress implies more effective units of labour available in the economy.*

- We express all variables in terms of effective units.

$$\tilde{y}_t \equiv \frac{Y}{AL} \tilde{k}_t \equiv \frac{Y}{AL}$$

Growth Rates

- Using constant returns to scale (CRS)

$$Y = F(K, AL) \;\Rightarrow\; \tilde{y}_t = f(\tilde{k}_t)$$

- Growth of capital per effective labour $\tilde{k}_t$:

$$\frac{\Delta \tilde{k}_t}{\tilde{k}_t} = \frac{\Delta K}{K} - \frac{\Delta L}{L} - \frac{\Delta A}{A}$$
$$= \frac{\Delta K}{K} - n - g$$

$$\frac{\Delta K}{K} = \frac{\Delta \tilde{k}_t}{\tilde{k}_t} + n + g$$

Solow: Deriving the Fundamental Equation

$$\overbrace{sY_t}^{\text{Saving}} = \overbrace{\Delta K_t + \delta K_t}^{\text{Investment}}$$

$$\frac{\Delta K_t}{K_t} = s \cdot \frac{Y_t}{K_t} - \delta \quad (2)$$

$$\frac{\Delta \tilde{k}_t}{\tilde{k}_t} = \frac{\Delta K_t}{K_t} - n - g \quad (3)$$

$$\frac{\Delta \tilde{k}_t}{\tilde{k}_t} = s\frac{\tilde{y}_t}{\tilde{k}_t} - (\delta + n + g) \quad \text{(FE-III)}$$

Fundamental Equation

Definition (Fundamental Equation)

$$\frac{\Delta \tilde{k}_t}{\tilde{k}_t} = s \cdot \frac{\tilde{y}_t}{\tilde{k}_t} - (\delta + n + g)$$

- *The growth rate of k_t depends*
  - *positively on s*
  - *positively on $\frac{Y_t}{K_t}$*
  - *negatively on δ*
  - *negatively on n*
  - *negatively on g*

Solow: Steady State

Definition (Steady State Condition)

$$\frac{\Delta k_t}{k_t} = s \cdot \frac{\tilde{y}_t}{\tilde{k}_t} - (\delta + n + g) = 0$$

$$\left[\frac{\tilde{y}^*}{\tilde{k}^*}\right] = \frac{\delta + n + g}{s}$$

- *The steady-state Output and Capital stock levels are*
  - *positively related with s*
  - *negatively related with δ*
  - *negatively related with n*
  - *negatively related with g*

Steady-State Growth

- Steady State:

$$\tilde{k}_t = \tilde{k}^*$$

- Constant $\tilde{k}_t$, capital per effective worker, implies

$$\frac{\Delta \tilde{k}_t}{\tilde{k}_t} = \frac{\Delta K}{K} - \frac{\Delta L}{L} - \frac{\Delta A}{A} = 0$$

- Capital per worker k grows at the rate g

$$\frac{\Delta k}{k} = g$$

Steady State Growth Path

- Similarly, $\tilde{y}_t$, output per effective worker, is constant

$$\tilde{y}_t = \tilde{y}^* \quad \Rightarrow \quad \frac{\Delta y}{y} = g$$

- We have finally got growth of k and y in steady state
  - Without technological progress, *capital accumulation runs into diminishing returns*
  - With technological progress, *improvements in technology continually offsets the diminishing returns to capital accumulation*

Solow: Convergence Dynamics

Proposition (Convergence Dynamics of Solow - II)

$$\frac{\Delta \tilde{k}_t}{\tilde{k}_t} = s\,\frac{\tilde{y}_t}{\tilde{k}_t} - (\delta + n + g) \qquad = s\left(\frac{\tilde{y}_t}{\tilde{k}_t} - \left[\frac{\tilde{y}^*}{\tilde{k}^*}\right]\right)$$

- *Further the economy is from the steady state, faster the growth rate of capital per worker k*
- *Higher the saving rate s, faster the economy converges to the steady state*

Summary

- With population growth (n) and technical progress (g), the Solow – III predicts that the economy's
  - √ capital stock and output grow at the rate $(n+g)$
  - √ capital stock per worker and output per worker grow at rate g
  - Solow - III gives us steady state growth of k, capital per worker and y, output per worker, at constant rate g for all economies

- Kaldor Facts: Empirically we observe variation in growth rate of output per worker across countries which Solow - III cannot explain very well, i.e.,
  - *poor countries do not necessarily grow faster than the rich ones*
- It does tell us where to look for an explanation ……

Effective Units of Labour

- AL: effective units of labour
- $\tilde{k}_t = \frac{K}{AL}$: capital stock per effective unit of labour
- $\tilde{y}_t = \frac{Y}{AL}$: output per effective unit of labour
- Fundamental Equation

$$\frac{\Delta \tilde{k}_t}{\tilde{k}_t} = s\frac{\tilde{y}_t}{\tilde{k}_t} - (\delta + n + g)$$

In convergence dynamics, saving does not exactly offset the reduction in $\tilde{k}_t$ attributable to depreciation, population growth and technological progress.

- Growth rate of $\tilde{k}_t$ (and $\tilde{y}_t$) determined by s, δ, n and g.

Steady State

- Steady State

$$\frac{\tilde{y}_t}{\tilde{k}_t} = \frac{\delta + n + g}{s}$$

In steady state, saving $sf(\tilde{k}_t)$ exactly offsets the reduction in $\tilde{k}_t$ attributable to depreciation, population growth and technological progress.

- Level of $\tilde{k}_t$ (and $\tilde{y}_t$) determined by s, δ, n and g

Sources of Growth

$$Y = F(K, AL) \qquad = K^{\alpha}(AL)^{1-\alpha} \qquad \text{(Cobb-Douglas)}$$

Y grows for three reasons:

1. Growth in K
2. Growth in L
3. Technological Progress (Growth in A)
    - Total Factor Productivity Growth (TFPG)
    - very difficult to measure

$$\frac{\Delta Y}{Y} = \alpha \frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L} + \underbrace{(1-\alpha)\frac{\Delta A}{A}}_{\text{TFPG}}$$

Capital income share

Proposition

Share of total income accruing to capital remains constant in steady state

- Share of capital income: $\dfrac{(r+\delta)\cdot K}{Y}$
  - $\dfrac{K}{Y}$ is constant (steady state property)
  - $r+\delta = f'(\tilde{k}_t)$ is constant (steady state property)

CAPITAL INCOME SHARE

Proposition

For Cobb-Douglas production function $Y = K^{\alpha}(AL)^{1-\alpha}$, the share of capital income and labour income is α and $(1-\alpha)$ respectively

- Show that for a Cobb Douglas production function

$$\frac{r+\delta \cdot K}{Y} = \alpha$$

  - $\frac{(r+\delta)\cdot K}{Y}$ is easy to measure.
  - For most developed economies, $\alpha \approx \frac{1}{3}$.

Total Factor Productivity

Definition (Total Factor Productivity Growth)

Growth of output that cannot be explained by growth of inputs. It is also called the Solow residual because it measures the residual growth and was first measured by Solow in 1957.

$$\text{TFPG} = \frac{\Delta Y}{Y} - \left(\frac{1}{3} \cdot \frac{\Delta K}{K} - \frac{1}{3} \cdot \frac{\Delta L}{L} \right)$$

- **US**: TFPG accounts for one-third of the growth
- **UK**: TFPG accounts for half of the growth

Summary

Two things are less than satisfactory with Solow Growth model

1. TFP is exogenous
   - we cannot explain exactly why we get growth in steady state
   - it does not tell us how to encourage growth
   - e.g. cannot explain the slowdown in the 70s
2. Global convergence of steady-state growth rate of output per capita to g
   - g is largely common knowledge
   - we do not observe this in practice

- Solow Model just tells us that we can solve the growth conundrum by looking for answers in technological progress.

1. Show the convergence dynamics of an economy which has a population growth rate n_1 and starting capital stock k_0.
2. The economy is at steady state B. Show what would happen if the economy's population growth increases to n_2.

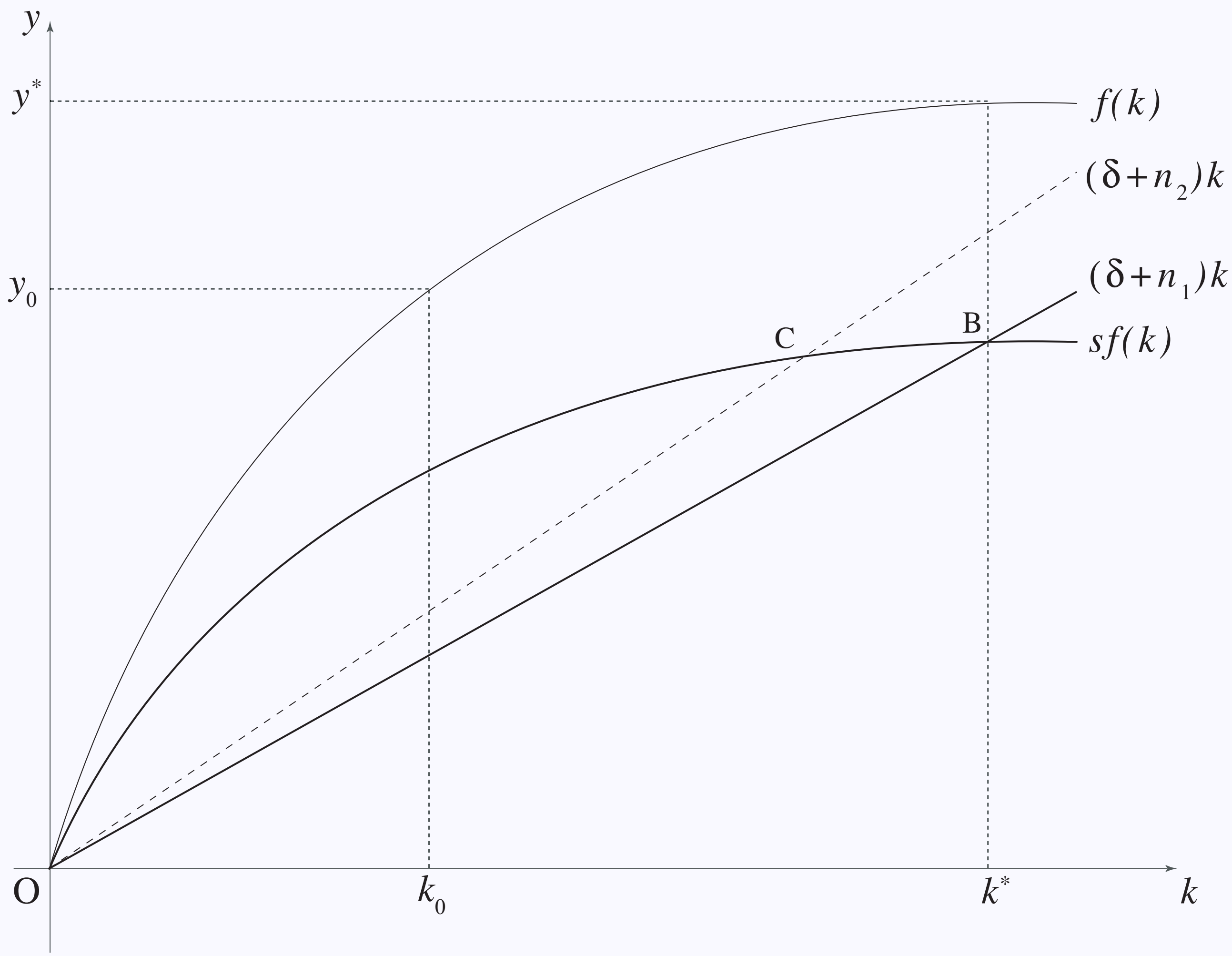


Figure: Increase in population growth rate and its effect on level of capital

1. Fill in the blanks in Figure. (*Hint: Put down the expression for growth rate of capital using the fundamental equation.*)

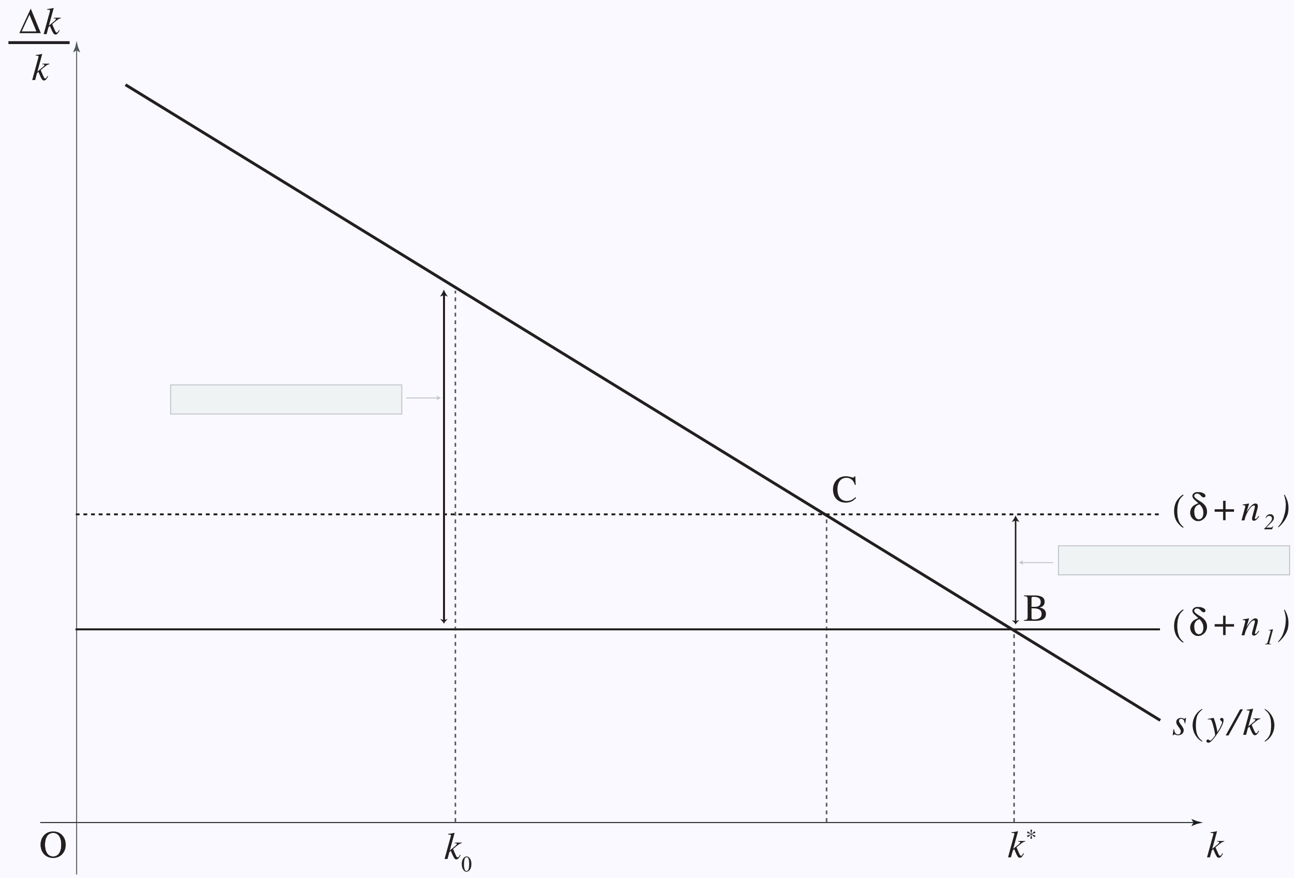


Figure: Increase in population growth rate and its effect capital growth rate

1. The economy is in steady state at A when the saving rate suddenly goes up from s_1 to s_2.
2. Show the convergence dynamics in each figure as the economy converges on to a new steady state.

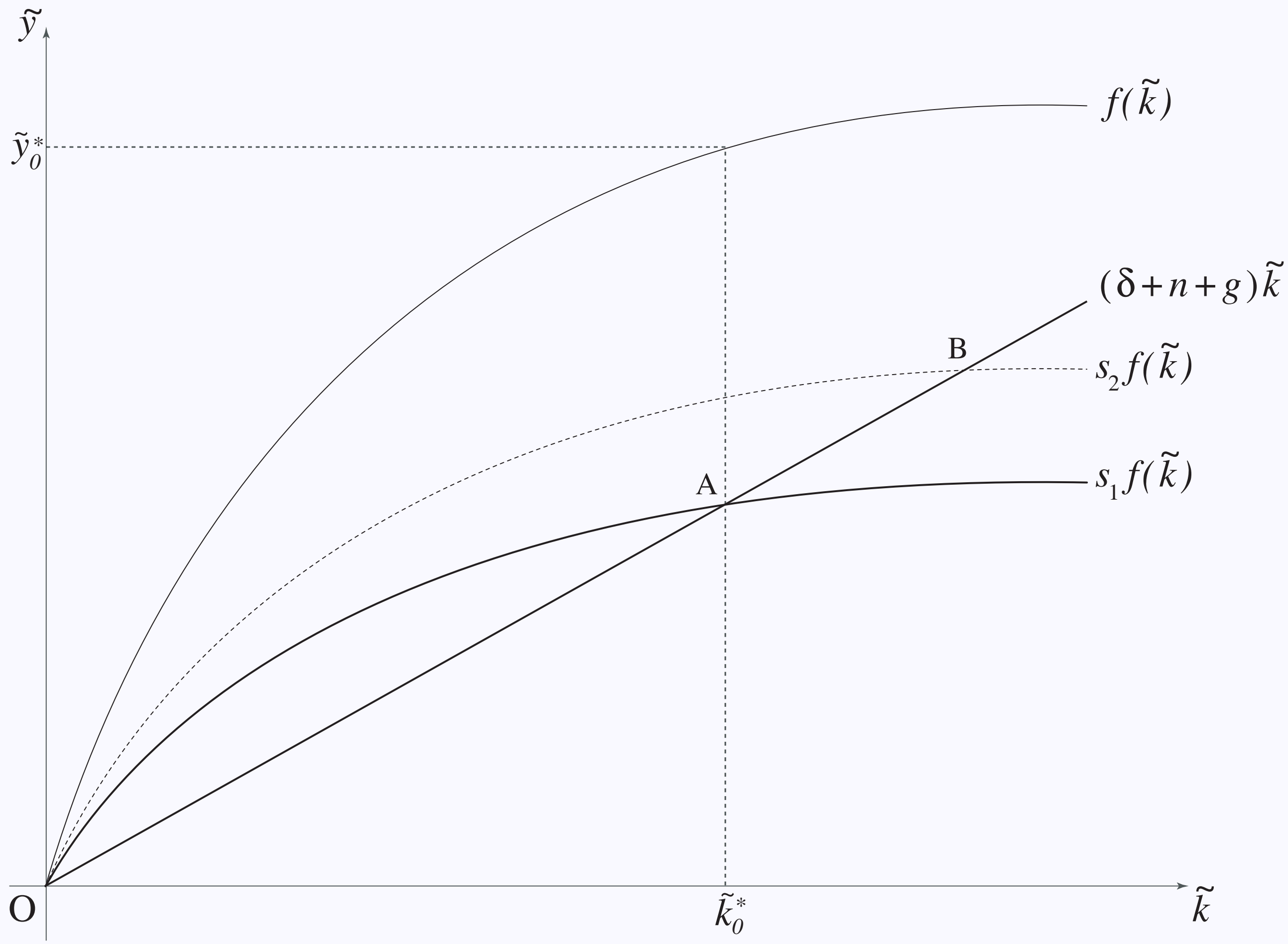


Figure: Convergence dynamics due to increase in saving rate

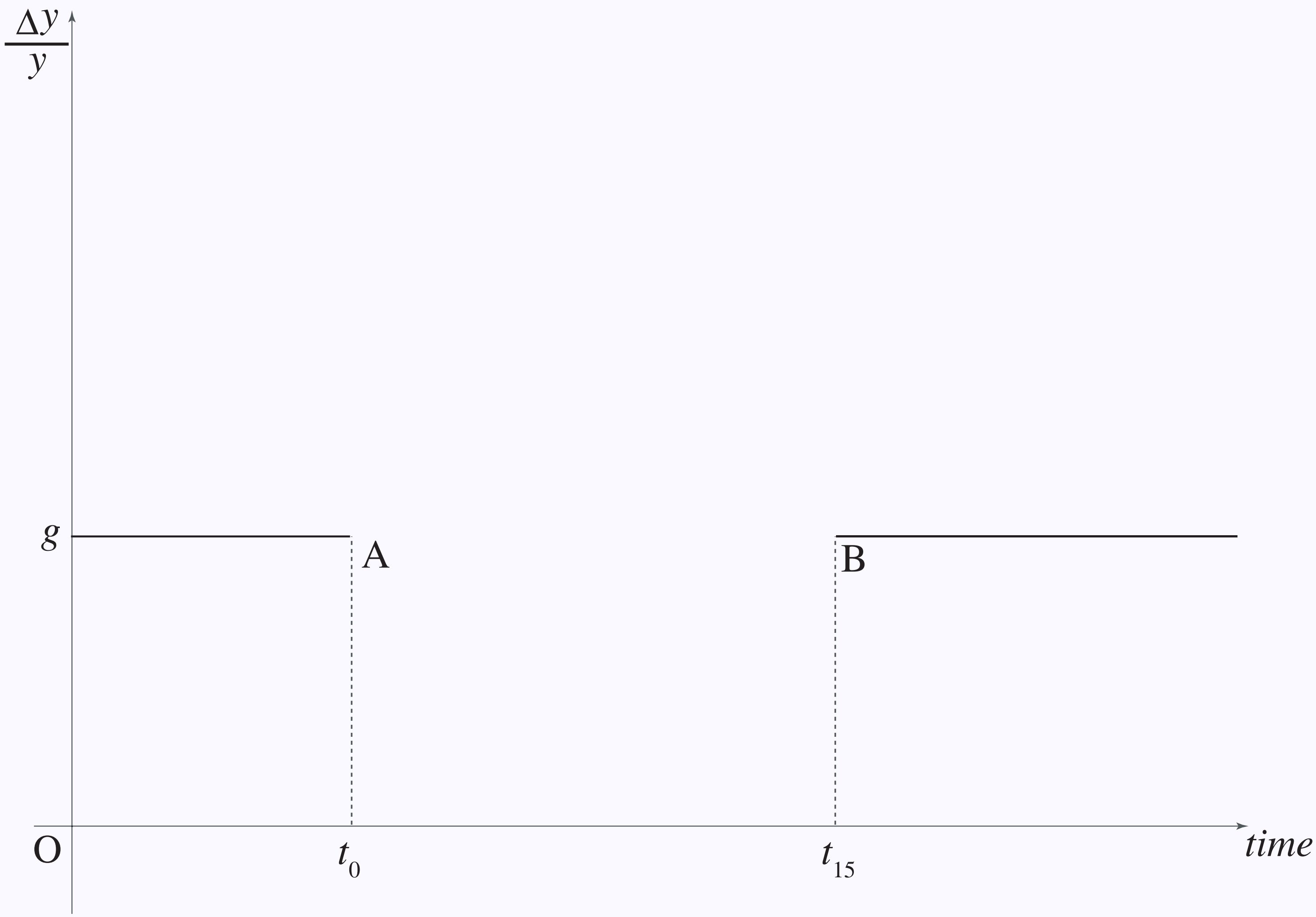


Figure: Growth of y due to increase in saving rate

Cobb Douglas production function

$$Y = K^{\alpha}(AL)^{1-\alpha}$$

Per-worker production function

$$y = A^{1-\alpha}k^{\alpha}$$

Determining the growth rate of y

$$\frac{\Delta y}{y} = (1-\alpha)\frac{\Delta A}{A} + \alpha\frac{\Delta k}{k}$$

Growth rates in Solow

$$\frac{\Delta y}{y} = (1-\alpha)\frac{\Delta A}{A} + \alpha\frac{\Delta k}{k}$$

1. $\frac{\Delta A}{A} = g$
2. $\frac{\Delta k}{k} = g$ (steady state)

$$\Rightarrow \quad \frac{\Delta y}{y} = (1-\alpha)g + \alpha g = g$$

- y grows at rate g in steady state because
  - A always grows at the rate g
  - k grows at the rate g in steady state

Externalities of Investment

Till now we have assumed that A grows at an exogenous rate g

Assumption (Positive Externalities of Investment)

The act of investment generates new ideas for both the investing firm and other firms in the economy. Specifically,

$$A = \lambda k \qquad (\lambda > 0)$$

stock of knowledge A is proportional to the stock of capital stock per worker k

- Investing $(k \uparrow)$ affects output y through two distinct channels:
  - **Direct effect**: greater capital stock per worker lead to greater output per worker
  - **Indirect effect**: higher capital stock per worker leads to higher value A which leads to higher output per worker.

The Two Channels

$$Y = K^{\alpha}(AL)^{1-\alpha} \qquad \text{(Cobb-Douglas)}$$

- Investing $(k\uparrow)$ affects output y through two distinct channels:
  - **Direct effect**: higher k leads to higher y
  - **Indirect effect**: higher k leads to higher value of A which leads to higher y.

$$y = (A)^{1-\alpha}k^{\alpha} \qquad = (\lambda k)^{1-\alpha}k^{\alpha} = \lambda^{1-\alpha}k$$

$$\Rightarrow \quad \frac{y}{k} = \lambda^{1-\alpha} = \text{ constant} \Rightarrow \qquad \frac{\Delta y}{y} = \frac{\Delta k}{k}$$

Deriving Endogenous Growth

$$\frac{\Delta K}{K} = s \cdot \frac{Y}{K} - \delta$$

$$\Rightarrow \quad \frac{\Delta k}{k} = s \cdot \frac{y}{k} - (\delta + n) \qquad = s \cdot \lambda^{1-\alpha} - (\delta + n)$$

- growth rate of k depends on the s, δ, n and λ

Endogenous Growth

$$\Rightarrow \quad \frac{\Delta y}{y} = s \cdot \lambda^{1-\alpha} - (\delta + n)$$

Perpetual growth of k and y if $s \cdot \lambda^{1-\alpha} > (\delta + n)$

- New Results:
  1. Steady state can only be defined in terms of growth rates. It cannot be defined in terms of levels anymore.
  2. Growth rate of y and k depends on δ, n, s and crucially on λ
     - Countries with higher λ grow faster. Explains why developed economies keep growing faster than certain under-developed. (due to higher λ)
     - Lower saving rate can lead to lower growth
     - Higher population growth rate can lead to slower growth. Paul Romer attributes slowdown in US growth in the 60s to this effect.

Causes of Growth

- Proximate versus **Fundamental** Causes of Growth

  Proximate Causes:

  - K, L, A, H

  Fundamental Causes:

  - The luck hypothesis
  - The geography hypothesis
  - The culture hypothesis
  - The institutions hypothesis

Summary: Endogenous Growth

- Externalities of capital investment create an extra channel through which investment affects the output
- Technological Progress is endogenised
  - Stock of Knowledge A is proportional to capital stock per worker k
- We get perpetual growth of k and y which depends on
  - s, δ, n
  - the strength of externality of capital investment, namely the value of λ
  - higher the value of λ, the faster the economy grows
- r is constant and w grows at the rate at which k and y grows

1. Draw the following production function below
   i. $y = A \cdot k^{\alpha}$
   ii. $y = \lambda^{1-\alpha} k$

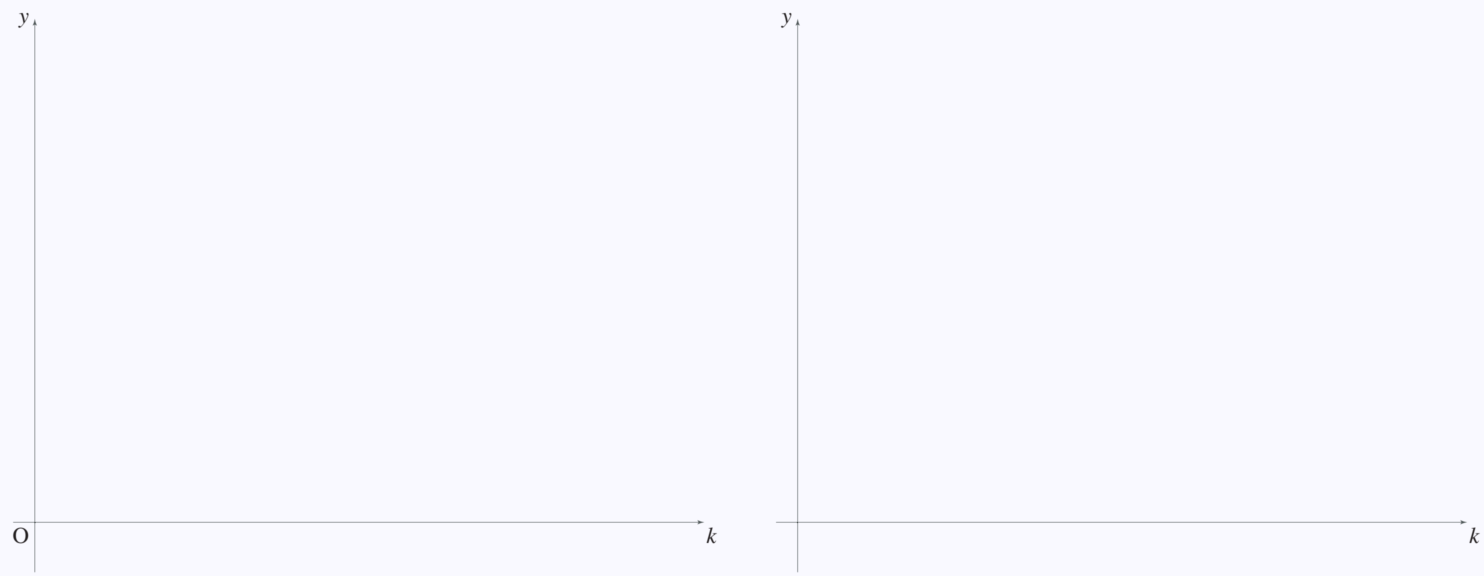


Figure: Production Functions

1. Trace the path of the economy if it starts from capital stock k_0.

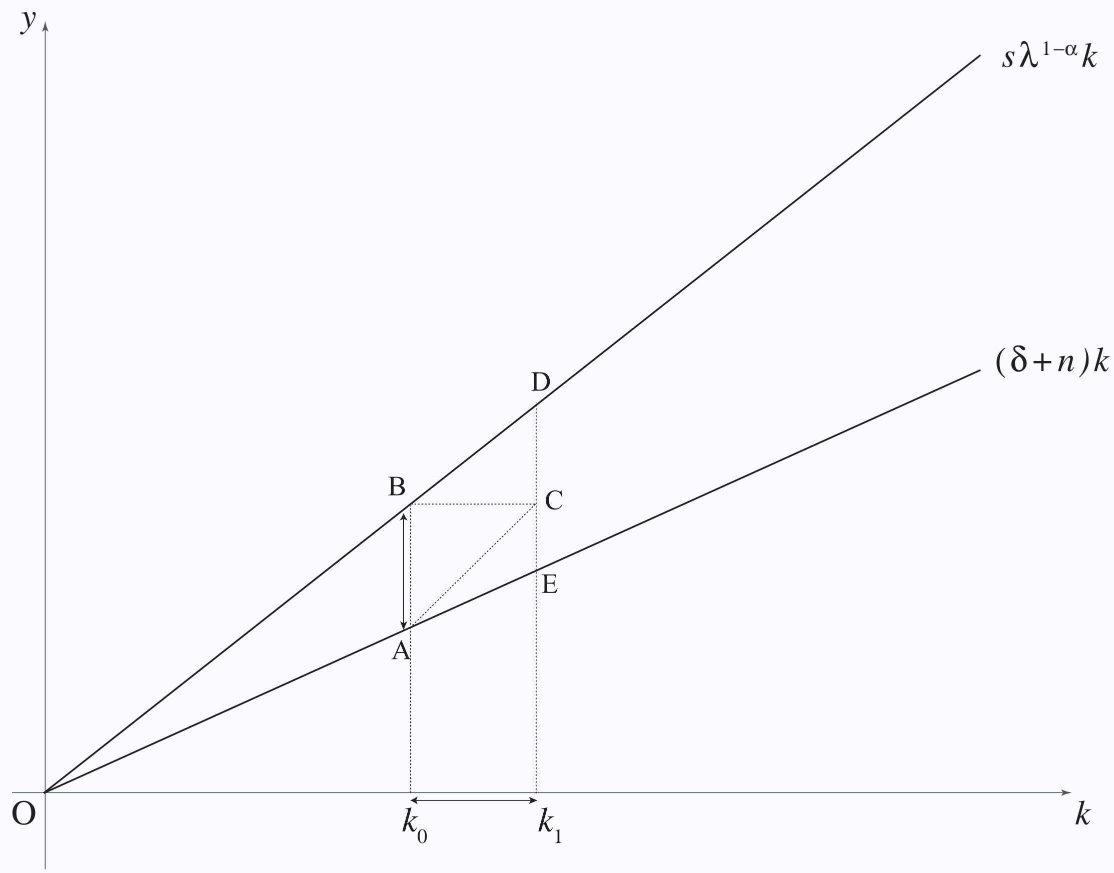
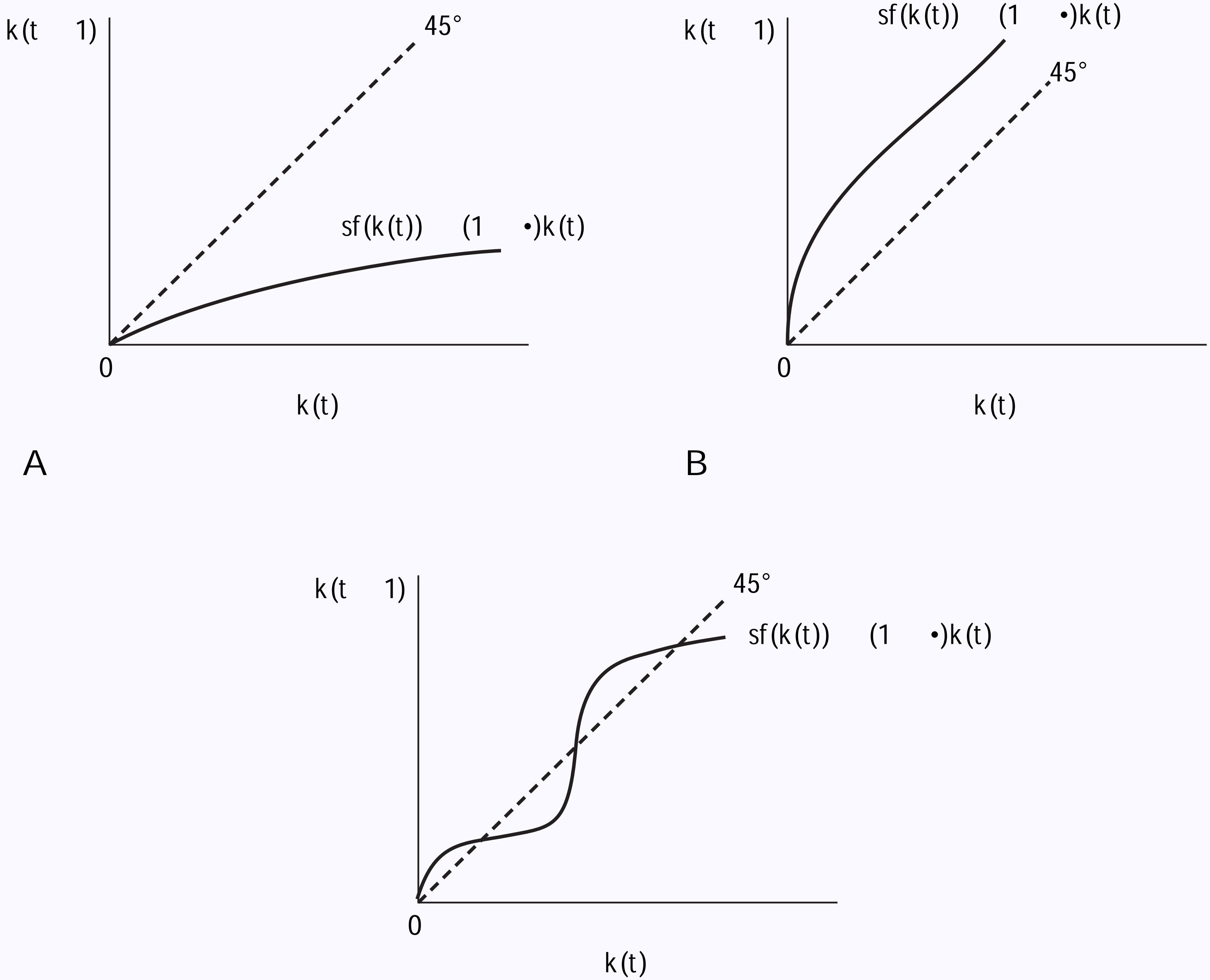


Figure: Endogenous growth with linear production function


k(t 1)
45°
sf(k(t)) (1 •)k(t)
0
k(t)
A
k(t 1)
sf(k(t)) (1 •)k(t)
45°
0
k(t)
B
k(t 1)
45°
sf(k(t)) (1 •)k(t)
0
k(t)